* Impedance Bridges & Q meters

D.C. and AC Bridges

Introduction –

* Bridge circuits are basically electrical Network
* They contain R.L.C as their circuit elements
* Used to measure unknown resistance (Low, medium, high) Inductance, Capacitance admittance, conductance etc.
* D.C. Bridges use dc energy source (i.e. Battery)
* A.C. bridges use AC energy source (i.e. normal A.C. power supply or oscillators)
* Bridges operate on the principle of null indication
* D.C. Bridge use current sensitive meters as null detector, preferably Galvanometer is used for this purpose.
* Each bridge has four arms
* for D.C. Bridges four arms are resistance arms
* for A.C. Bridge four arms are generally impedance arms

(Note :- one or two arms out of four for A.C. bridges may be purely resistive)

⇒ * D.C. bridges are basically used to measure unknown resistances of low, medium and high values

* Low resistance (i.e. $R_{low} \leq 1\Omega$)
* Medium resistance (i.e. $1\Omega \leq R_{med} \leq 10^5\,\Omega$
* High resistance (i.e. $R_{high} \geq 10^5\,\Omega$)

⇒ * A.C. Bridges are used to measure unknown inductance, capacitance, impedance, admittance etc.

WHEATSTONE BRIDGE (MEASUREMENT OF R)

Wheatstone bridge is the most accurate method available for measuring resistance and is popular for laboratory use.
The ckt diagram of Wheatstone bridge is given in fig.

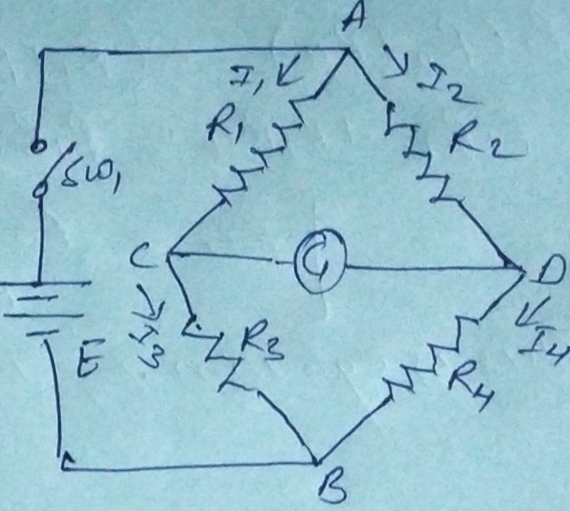


fig. wheatstones bridge

The source of emf and indicating meter, the galvanometer, is connected to point C & D. The galvanometer is a sensitive micro ammeter a zero center scale, when there is no current through the meter, the galvanometer points rest at 0 i.e. mid scale, current in one direction causes the points deflect on one side and current in the opposite direction to the other side

when SW_1 is closed, current flows and divides into the two arms at point A i.e I_1 & I_2. The bridge is balanced when there is no current through the galvanometer, or when the potential difference at point C & D is equal, i.e. the potential across the galvanometer is zero

To obtain the bridge balance equation (from fig)

for the galvanometer current to be zero the following condition should be satisfied.

$$\frac{\cancel{E} \times R_1}{R_1 + R_3} = \frac{\cancel{E} \times R_2}{R_2 + R_4}$$

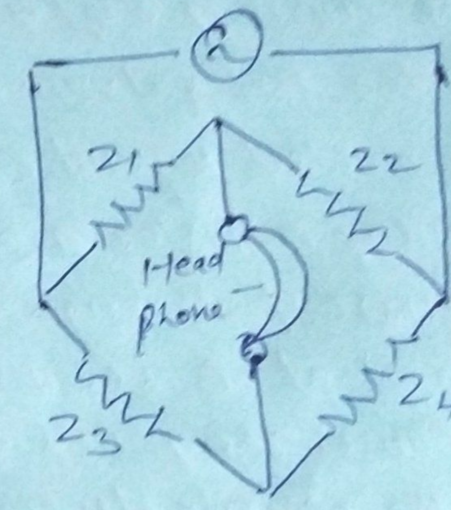
$$R_1(R_2 + R_4) = (R_1 + R_3)R_2$$

$$\cancel{R_1 R_2} + R_1 R_4 = \cancel{R_1 R_2} + R_3 R_2$$

$$\left[R_4 = \frac{R_2 R_3}{R_1} \right]$$

This is equation for the bridge to be balanced.

AC Bridges (Impedance)



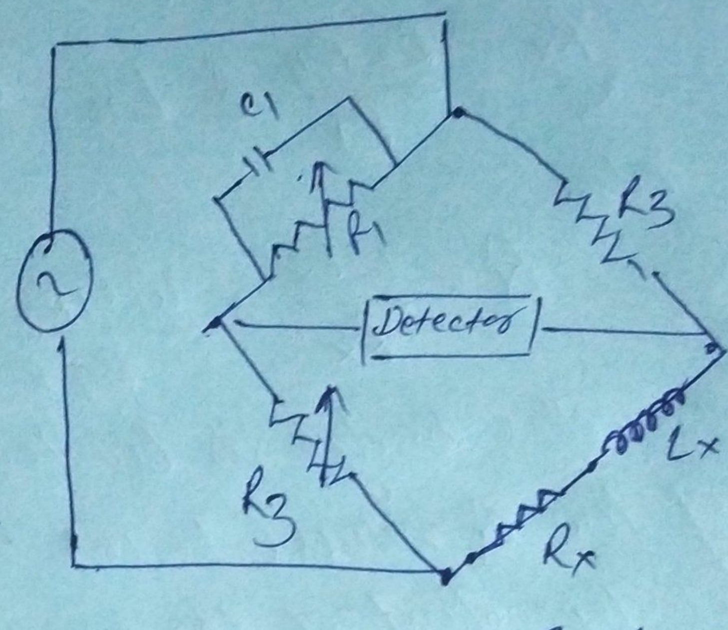
AC. Wheatstone bridge.

Impedances at AF or RF are commonly determined by means of an ac wheatstone bridge. This bridge is similar to a dc bridge, except that the bridge arms are impedances, The bridge is excited by an ac source rather then dc & the galvanometer is replace by a detector such as a pair of headphones for detecting ac. when the bridge is balanced

$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}$$

where Z_1, Z_2, Z_3 & Z_4 are the impedance of the arms and are vector complex quantities that possess phase angles, It is thus necessary to adjust both the magnitude and phase angles of the impedance arms to achieve balance i.e. the bridge must be balanced for both the reactance and the resistive components

Maxwell's Bridge or maxwell's Induction Bridge



Maxwell's Bridge.

maxwell's Bridge measures an unknown inductance in of standard arm offers the advantage of compactness and easy shielding. The capacitor is almost a loss-less component. one arm has a resistance R_1 in parallel with C_1 and hence it is easier to write the balance equation using the admittance of arm 1 instead of the impedance.

The general equation for bridge balance is

$$Z_1 Z_x = Z_2 Z_3$$

$$Z_x = \frac{Z_2 Z_3}{Z_1} = Z_2 Z_3 Y_1 \quad — ①$$

$Z_1 = R_1$ in parallel with C_1 i.e. $Y_1 = \frac{1}{Z_1}$

$$Y_1 = \frac{1}{R_1} + j\omega C_1$$

$Z_2 = R_2$, $Z_3 = R_3$, $Z_x = R_x$ in series with L_x

$$Z_x = R_x + j\omega L_x$$

from equa of Z_x we get eq ①

$$R_x + j\omega L_x = R_2 R_3\left(\frac{1}{R_1} + j\omega C_1\right)$$

$$R_x + j\omega L_x = \frac{R_2 R_3}{R_1} + j\omega C_1 R_2 R_3$$

Equating real terms & Imaginary terms we have

$$R_x = \frac{R_2 R_3}{R_1} \quad \& \quad L_x = C_1 R_2 R_3$$

also

$$Q = \frac{\omega L_x}{R_x} = \frac{\omega C_1 R_2 R_3 \times R_1}{R_2 R_3} = \boxed{\omega C_1 R_1}$$

maxwell's bridge is limited to the measurment of low Q values (1–10), the measurement is independent of the excitation frequency. the scale of the resistance can be calibrated to read inductance directly.

Hay's Bridge

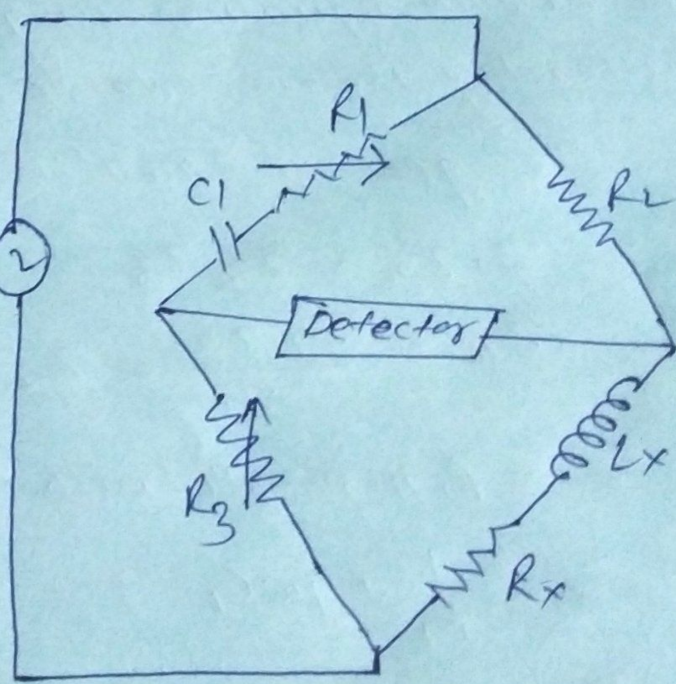


fig – Hay's Bridge

The Hay's bridge differs from maxwell's bridge by having a resistance R_1 in series with a standard capacitor C_1 instead of a parallel, for large phase angles, R_1 needs to be low, therefore this bridge is more convenient for measuring high Q coils. for Q = 10 the error is ± 1%

Q = 30 the error is ± 0.1%

Hence Hay's bridge is preferred for coil with a high Q & Maxwell's bridge for coil with a low Q.

For Hays Bridge

At Balance $Z_1 Z_x = Z_2 Z_3$

$Z_1 = R_1 - j/\omega C_1$

$Z_2 = R_2$

$Z_3 = R_3$

$Z_x = R_x + j\omega L_x$

Substituting these values in the balance equation we get

$$\left(R_1 - \frac{j}{\omega C_1}\right)(R_x + j\omega L_x) = R_2 R_3$$

$$R_1 R_x + \frac{L_x}{C_1} - \frac{jR_x}{\omega C_1} + j\omega L_x R_1 = R_2 R_3$$

Equating the real & imaginary terms we have

$$R_1 R_x + \frac{L_x}{C_1} = R_1 R_3 \quad — ① \text{ (Really)}$$

$$\frac{R_x}{\omega C_1} = \omega L_x R_1 \quad — ② \quad \text{(Imaginary)}$$

Solving for L_x and R_x we have $R_x = \omega^2 L_x C_1 R_1$

Substituting for R_x in equation ①

$$R_1(\omega^2 R_1 C_1 L_x) + \frac{L_x}{C_1} = R_2 R_3$$

$$\omega^2 R_1^2 C_1 L_x + \frac{L_x}{C_1} = R_2 R_3$$

multiply both sides by C_1 we get

$$\omega^2 R_1^2 C_1^2 L_x + L_x = R_2 R_3 C_1$$

Therefore $$\left[L_x = \frac{R_3 R_2 C_1}{1 + \omega^2 R_1^2 C_1^2}\right]$$

Substituting for L_x in eq (2) we get

$$\left[R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1+\omega^2 R_1^2 C_1^2} \right]$$

The term ω appears in the expression for both L_x and R_x. This indicates that the bridge is frequency sensitive.

[Wien Bridge →

Wien Bridge has a combination in one series RC and a parallel combination in the adjoining arm. Wien bridge in its basic form is designed to measure freq. It can also be used for the instrument of an unknown capacitor with great accuracy.

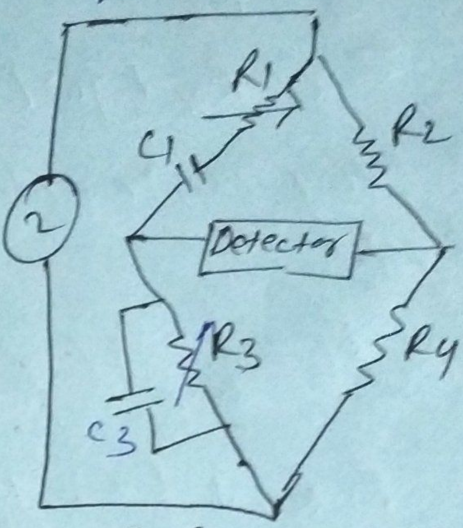


fig. (Wein Bridge)

The impedance of one series arm is $Z_1 = R_1 - j/\omega C_1$

The admittance of the parallel arm is - $Y_3 = 1/R_3 + j\omega C_3$

using the Bridge Balance equation we have

$$Z_1 Z_4 = Z_2 Z_3$$

Therefore - $Z_1 Z_4 = Z_2 / Y_3$ i.e. $Z_2 = Z_1 Z_4 Y_3$

$$R_2 = R_4 \left(R_1 - \frac{j}{\omega C_1} \right)\left(\frac{1}{R_3} + j\omega C_3 \right)$$

$$R_2 = \frac{R_1 R_4}{R_3} - \frac{j R_4}{\omega C_1 R_3} + j\omega C_3 R_1 R_4 + \frac{C_3 R_4}{C_1}$$

$$R_2 = \left(\frac{R_1 R_4}{R_3} + \frac{C_3 R_4}{C_1} \right) - j\left(\frac{R_4}{\omega C_1 R_3} - \omega C_3 R_1 R_4 \right)$$

$$R_2 = \frac{R_1 R_4}{R_3} + \frac{C_3 R_4}{C_1} \text{ (Real)} \quad \frac{R_4}{\omega C_1 R_3} - \omega C_3 R_1 R_4 = 0 \text{ (Imaginary)}$$

Therefor

$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1} \quad \text{—} (1)$$

and

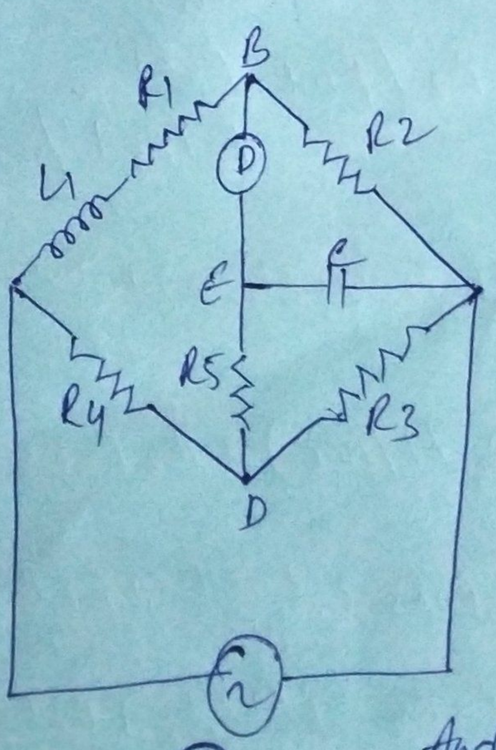
$$\frac{1}{\omega C_1 R_3} = \omega C_3 R_1 \quad \text{—} (2)$$

$$\omega^2 = \frac{1}{C_1 R_1 R_3 C_3} \Rightarrow \omega = \frac{1}{\sqrt{C_1 R_1 C_3 R_3}} \qquad \because \omega = 2\pi f$$

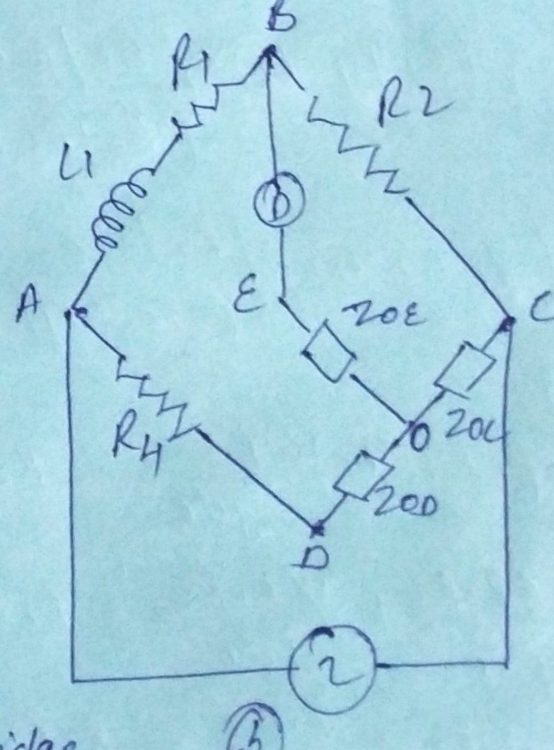
$$f = \frac{1}{2\pi\sqrt{C_1 R_1 C_3 R_3}} \quad \text{—} (3)$$

The two condition for bridge balance (1) & (3), result in an expression determining the required resistance Ratio R_2/R_4 and another express determining the frequency of The applied voltage, if we satisfy Eq (1) an also excite the bridge with the freq of eq (3) the bridge will be Balanced

⊛ ANDERSONS Bridge →



(a)



(b)

Andersson's Bridge

Anderson Bridge is a very important and useful modification of the Maxwell's – Wein Bridge as shown in fig (a)

The Balanced Condition for this bridge can be easily obtained by converting the mesh impedance C, R_3, R_5 to a equivalent star with the star point O as shown in fig (b) by using star/delta transformation

As per delta to star transformation –

$$Z_{OD} = \frac{R_3 R_5}{(R_3 + R_5 + 1/j\omega C)} \qquad Z_{OC} = \frac{R_3/j\omega C}{(R_3 + R_5 + 1/j\omega C)} = Z_3$$

Hence with reference to fig (b) it can be seen that

$$Z_1 = (R_1 + j\omega L_1),\ Z_2 = R_2,\ Z_3 = Z_{OC} = \frac{R_3/j\omega C}{(R_3 + R_5 + 1/j\omega C)}$$

$$Z_4 = R_4 + Z_{OD}$$

for balance bridge condition

$$Z_1 Z_3 = Z_2 Z_4$$

therefor – $(R_1 + j\omega L_1) \times Z_{OC} = Z_2 \times (Z_4 + Z_{OD})$

$$(R_1 + j\omega L_1) \times \left(\frac{R_3/j\omega C}{R_3 + R_5 + 1/j\omega C}\right) = R_2\left(R_4 + \frac{R_3 R_5}{R_3 + R_5 + 1/j\omega C}\right)$$

$$(R_1 + j\omega L_1) \times \left(\frac{R_3/j\omega C}{\cancel{R_3 + R_5 + 1/j\omega C}}\right) = R_2\left(R_4(R_3 + R_5 + 1/j\omega C) + \frac{R_3 R_5}{\cancel{(R_3 + R_5 + 1/j\omega C)}}\right)$$

$$(R_1 + j\omega L_1) \times \frac{R_3}{j\omega C} = R_2 R_4 (R_3 + R_5 + 1/j\omega C) + R_2 R_3 R_5$$

$$\frac{R_1 R_3}{j\omega C} + \frac{j\omega L_1 R_3}{j\omega C} = R_2 R_3 R_4 + R_2 R_4 R_5 + \frac{R_2 R_4}{j\omega C} + R_2 R_3 R_5$$

$$\frac{-jR_1 R_3}{\omega C} + \frac{L_1 R_3}{C} = R_2 R_3 R_4 + R_2 R_4 R_5 - j\frac{R_2 R_4}{\omega C} + R_2 R_3 R_5$$

Equating the real terms and imaginary terms

$$\frac{L_1 R_3}{C} = R_2 R_3 R_4 + R_2 R_4 R_5 + R_2 R_3 R_5 \text{ (real)}$$

$$L_1 = \frac{C}{R_3}\left(R_2 R_3 R_4 + R_2 R_4 R_5 + R_2 R_3 R_5\right)$$

$$L_1 = C R_2\left(R_4 + \frac{R_4 R_5}{R_3} + R_5\right)$$

$$L_1 = C R_2\left[R_4 + R_5 + \frac{R_4 R_5}{R_3}\right]$$

$$\frac{-j R_1 R_3}{\omega C} = \frac{-j R_2 R_4}{\omega C} \text{ (Imaginary)}$$

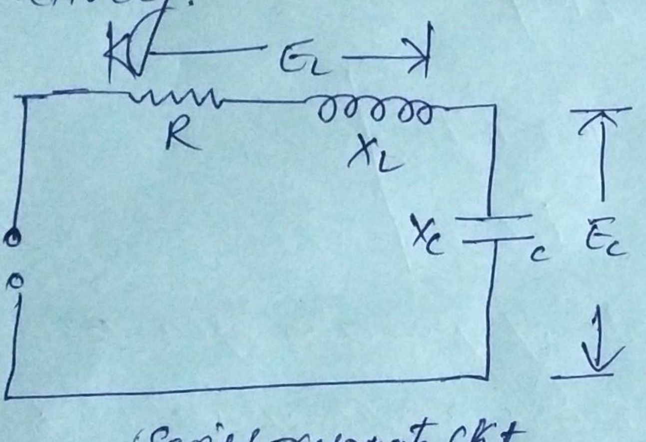
$$R_1 R_3 = R_2 R_4$$

and Therfor $\left[R_1 = \frac{R_2 R_4}{R_3}\right]$

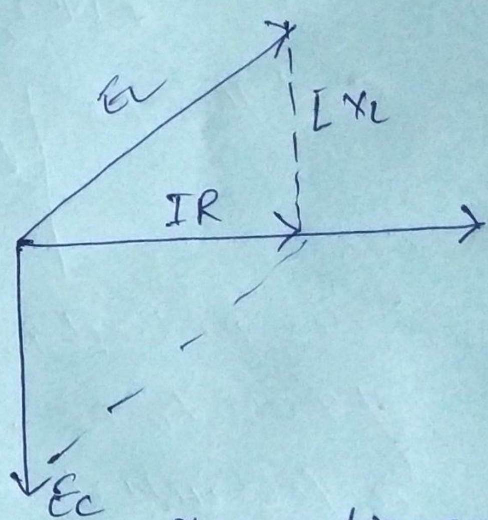
This method is capable of precise measurement of inductance and a wide range of value from a few uH to several Henry.

Q meter ->

The instrument which measure some of the electrical properties of coils and capacitors is referred as Q-meter. The working principle of Q-meter depends on the characteristics of a resonance circuits, i.e the voltage drope across the coil or capacitor is equal to the applied voltage times the Q-factor of the circuit. Thus if the circuit subjected to a fixed voltage, the voltmeter connected across the capacitor is calibrated to indicate the Q value directly. A series resonance ckt and it's voltage and current relationship at resonance condition are illustrated in fig. respectively.



Series resonate ckt



phasor diagram

At resonance condition

$$X_L = X_C$$

$$E_C = IX_L = IX_C$$

$$E = IR$$

where - X_C = capacitive reactance
X_L = Inductive reactance
I = current flowing through the ckt
E = Applied voltage, R = Resistance of the coil

The Q-factor or the magnification of the ckt is defined as.

$$Q = \frac{X_L}{R} = \frac{X_C}{R} = \frac{E_C}{E}$$

from the above equation it is clear that if the voltage E is maintained at a fixed level, the voltmeter across the capacitor can be calibrated in terms of Q directly. The ckt arrangement of basic and practical Q-meter is shown below.

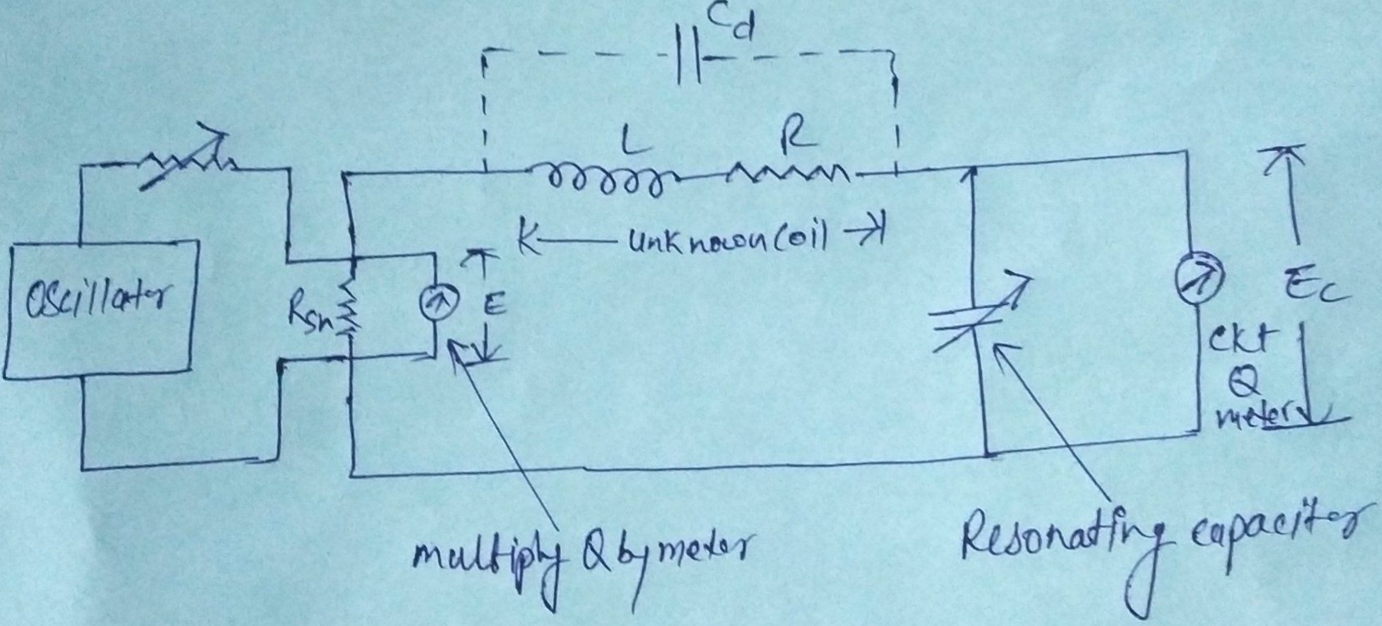


fig. Basic Q meter.

The oscillator is a wide range RF oscillator that supplies the oscillations whose freq. lies b/w 50KHz to 50MHz. and delivers current to R_{SH} which is a shunt resistance of low value, and is typically around 0.02 Ω. Therefore the R_{sh} introduces very negligible (almost no resistance) resistance into the oscillator circuit; Thus it represents a voltage source of magnitude E with very low internal resistance.

The voltage across R_{sh} is measured using a thermo-couple meter that is marked as multiply Q by meter. The voltage drop across the tuning capacitor or resonating capacitor E_c is measured by mean of an electronic voltmeter, The scale of this electronic voltmeter is calibrated in terms of Q values directly