CHAPTER

7
Two-port Network

7.1 INTRODUCTION

A ***port*** is a pair of nodes across which a device can be connected. The voltage is measured across the pair of nodes and the current going into one node is the same as the current coming out of the other node in the pair. These pairs are entry (or exit) points of the network.

So, a network with two input terminals and two output terminals is called a ***four-terminal network*** or a ***two-port network***.

It is convenient to develop special methods for the systematic treatment of networks. In the case of a single-port linear active network, we obtained the Thevenin's equivalent circuit and the Norton's equivalent circuit. When a linear passive network is considered, it is convenient to study its behaviour relative to a pair of designated nodes.

In a two-port network, there are two voltage variables and two current variables. According to the choice of input and output port, these voltage and current variables can be arranged in different equations, giving rise to different port parameters.

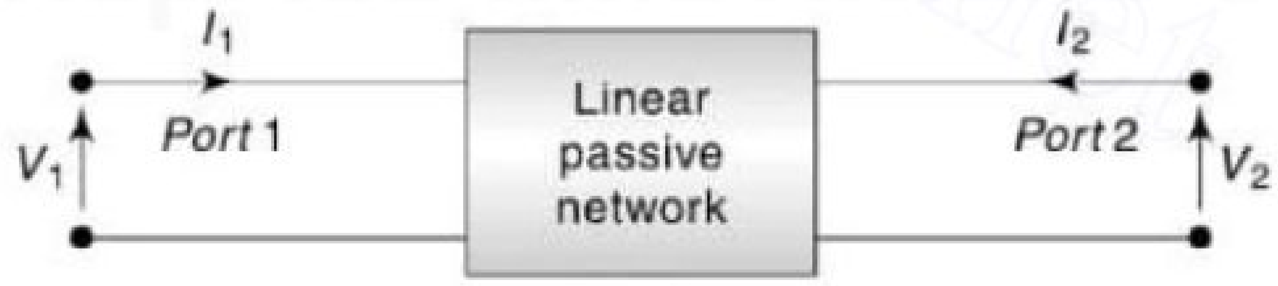


Figure 7.1 *Block diagram of a two-port network*

In this chapter, we will discuss the behaviours of two-port networks.

7.2 RELATIONSHIPS OF TWO-PORT VARIABLES

In order to describe the relationships among the port voltages and currents of an *n*-port network, '*n*' number of linear equations is required. However, the choice of two independent and two dependent variables is dependent on the particular application.

For n-port network, the number of voltage and current variables is $2n$. The number of ways in which these $2n$ variables can be arranged in two groups of n each is $\frac{2n!}{n! \times n!} = \frac{2n!}{(n!)^2}$. So, there will be $\frac{2n!}{(n!)^2}$ types of port parameters.

For a two-port network ($n = 2$), there are six types of parameters as mentioned below:—

1. Open-Circuit Impedance Parameters (z-parameters),
2. Short-Circuit Admittance Parameters (y-parameters),
3. Transmission or Chain Parameters (T- parameters or $ABCD$ – parameters),
4. Inverse Transmission Parameters (T'-parameters),
5. Hybrid Parameters (h-parameters), and
6. Inverse Hybrid Parameters (g-parameters).

Note: Inverse parameters (T' and g) are not included in WBUT syllabus.

7.2.1 Open-Circuit Impedance Parameters (z-parameters)

The impedance parameters represent the relation between the voltages and the currents in the two-port network.

The impedance parameter matrix may be written as,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \text{or} \quad \begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned}$$

In this matrix equation, it is easily seen without even expanding the individual equations, that

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \text{Driving Point Impedance at Port-1.}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \text{Transfer Impedance}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \text{Transfer Impedance}$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \text{Driving Point Impedance at Port-2}$$

It can be seen that the z-parameters correspond to the *driving point* and *transfer* impedances at each port with the other port having zero current (i.e. open circuit). Thus these parameters are also referred to as the open circuit parameters.

7.2.2 Short-Circuit Admittance Parameters (y-parameters)

The admittance parameters represent the relation between the currents and the voltages in the two-port network.

The admittance parameter matrix may be written as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{or} \quad \begin{matrix} I_1 = y_{11}V_1 + y_{12}V_2 \\ I_2 = y_{21}V_1 + y_{22}V_2 \end{matrix}$$

The parameters y_{11}, y_{12}, y_{21}, y_{22} can be defined in a similar manner, with either V_1 or V_2 on short circuit.

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \text{Driving Point Admittance at Port-1}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \text{Transfer Admittance}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \text{Transfer Admittance}$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \text{Driving Point Admittance at Port-2}$$

It can be seen that the y-parameters correspond to the *driving point* and *transfer* admittances at each port with the other port having zero voltage (i.e., short circuit). Thus these parameters are also referred to as the short circuit parameters.

7.2.3 Transmission Line Parameters (ABCD-parameters)

The *ABCD* parameters represent the relation between the input quantities and the output quantities in the two-port network. They are thus voltage-current pairs.

However, as the quantities are defined as an input-output relation, the output current is marked as going out rather than as coming into the port.

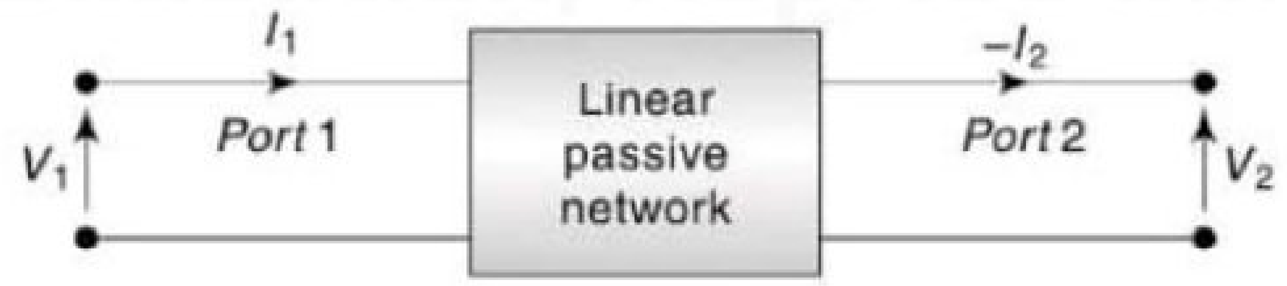


Figure 7.2 *Two-port current and voltage variables for calculation of transmission line parameters*

The transmission parameter matrix may be written as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad \text{or} \quad \begin{matrix} V_1 = AV_2 - BI_2 \\ I_1 = CV_2 - DI_2 \end{matrix}$$

The parameters A, B, C, D can be defined in a similar manner with either port 2 on short circuit or port 2 on open circuit.

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \text{Open Circuit Reverse Voltage Gain}$$

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0} = \text{Short Circuit Transfer Impedance}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \text{Open Circuit Transfer Admittance}$$

$$D = -\left.\frac{I_1}{I_2}\right|_{V_2=0} = \text{Short Circuit Reverse Current Gain}$$

These parameters are known as transmission parameters as in a transmission line, the currents enter at one end and leaves at the other end, and we need to know a relation between the sending end quantities and the receiving end quantities.

7.2.4 Hybrid Parameters (*h*-parameters)

The hybrid parameters represent a mixed or hybrid relation between the voltages and the currents in the two-port network.

The hybrid parameter matrix may be written as

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \quad \text{or} \quad \begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned}$$

The h-parameters can be defined in a similar manner and are commonly used in some electronic circuit analysis.

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \text{Short Circuit Impedance at Port-1}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \text{Open Circuit Reverse Voltage Gain}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \text{Short Circuit Current Gain}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \text{Open Circuit Output Admittance}$$

As the h-parameters are dimensionally mixed, they are also named mixed parameters. Transistor circuit models are generally represented by these parameters as the input impedance (h_{11}) and the short-circuit current gain (h_{21}) can be easily measured by making the output short-circuited.

7.3 CONDITIONS FOR RECIPROCITY AND SYMMETRY

A network is said to be reciprocal if the ratio of the response transform to the excitation transform is invariant to an interchange of the positions of the excitation and response of the network.

A two-port network will be reciprocal if the interchange of an ideal voltage source at one port with an ideal current source at the other port does not alter the ammeter reading.

A two-port network is said to be symmetrical if the input and output ports can be interchanged without altering the port voltages and currents.

1. Conditions in terms of z-parameters

Condition for Reciprocity We short circuit port 2 – 2′ and apply a voltage source V_s at port 1 – 1′.

Therefore, $V_1 = V_s$, $V_2 = 0$, $I_2 = -I_2'$

Writing the equations of z-parameters,

$$V_s = z_{11}I_1 - z_{12}I_2'$$

$$0 = z_{21}I_1 - z_{22}I_2'$$

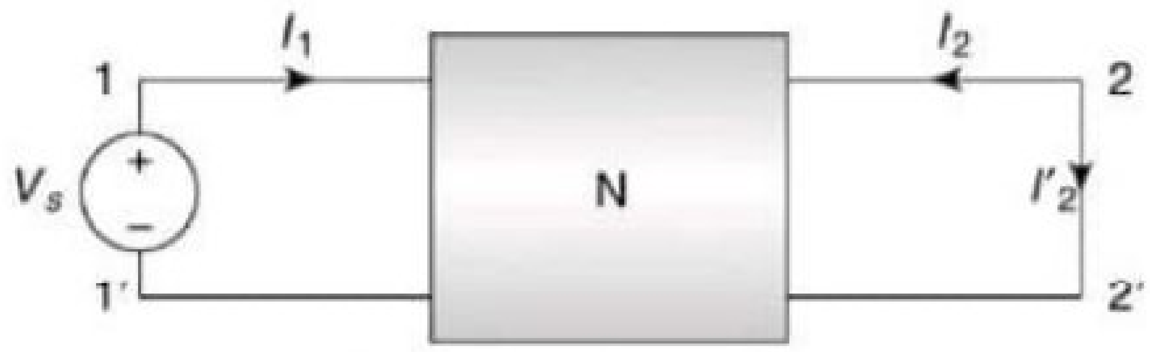


Fig. 7.3(a) *Reciprocal network*

Solving these two equations for I_2',

$$I_2' = V_s \frac{z_{21}}{z_{11}z_{22} - z_{12}z_{21}} \tag{7.1}$$

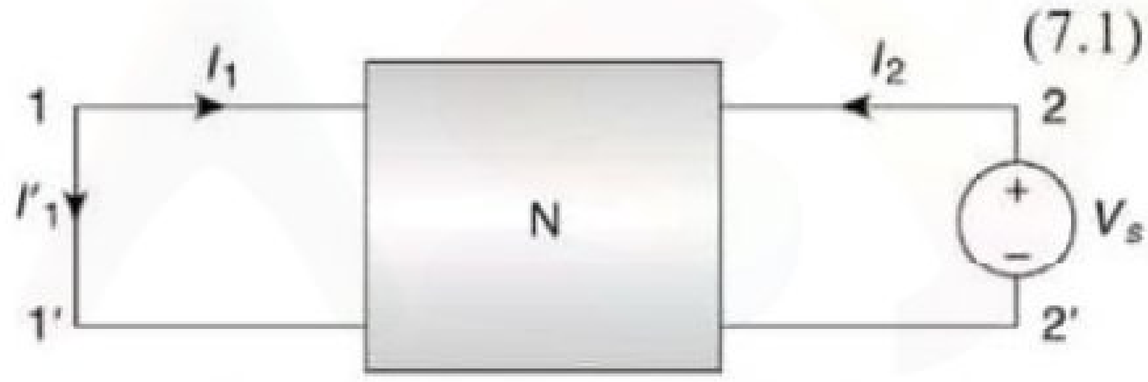


Fig. 7.3(b) *Reciprocal network*

Now, interchanging the positions of response and excitations, i.e., shorting port 1 – 1′ and applying V_s at port 2 – 2′; $V_1 = 0$, $V_2 = V_s$, $I_1 = I_1'$

Writing the equations of z-parameters,

$$0 = -z_{11}I_1' + z_{12}I_2$$

$$V_s = -z_{21}I_1' + z_{22}I_2$$

Solving these two equations for I_1',

$$I_1' = V_s \frac{z_{12}}{z_{11}z_{22} - z_{12}z_{21}} \tag{7.2}$$

For the two-port network to be reciprocal, from Eq. (7.1) and Eq. (7.2), we have the condition as,

$$\boxed{\mathbf{z_{12} = z_{21}}}$$

Condition for Symmetry

Applying a voltage V_s at port 1 – 1′ with port 2 – 2′ open, we have the equation,

$$V_s = z_{11}I_1 - z_{12}\cdot 0 = z_{11}I_1 \quad \Rightarrow \quad \left.\frac{V_s}{I_1}\right|_{I_2=0} = z_{11} \tag{7.3}$$

Now, applying a voltage V_s at port 2 – 2′ with port 1 – 1′ open, we have the equation,

$$V_s = z_{21}\cdot 0 + z_{22}I_2 = z_{22}I_2 \quad \Rightarrow \quad \left.\frac{V_s}{I_2}\right|_{I_1=0} = z_{22} \tag{7.4}$$

For the network to be symmetrical, the voltages and currents should be same. From Eq. (7.3) and Eq. (7.4), we have the condition for symmetry as,

$$\boxed{\mathbf{z_{11} = z_{22}}}$$

2. Conditions in terms of y-parameters

Condition for Reciprocity

From Fig. 7.3(a), writing the y-parameter equations,

$$\begin{aligned} I_1 &= y_{11}V_s \\ -I_2' &= y_{21}V_s \end{aligned} \quad \Rightarrow \quad -\frac{I_2'}{V_s} = y_{21} \tag{7.5}$$

From Fig. 7.3(b), writing the y-parameter equations,

$$\begin{aligned} -I_1' &= y_{12}V_s \\ I_2 &= y_{22}V_s \end{aligned} \quad \Rightarrow \quad -\frac{I_1'}{V_s} = y_{12} \tag{7.6}$$

From the principle of reciprocity, the condition for reciprocity is,

$$\boxed{\mathbf{y_{12} = y_{21}}}$$

Condition for Symmetry

As already stated, a two-port network is said to be symmetric if the ports can be interchanged without changing the port voltages and currents and thus the condition of symmetry becomes,

$$\boxed{\mathbf{y_{11} = y_{22}}}$$

3. Conditions in terms of ABCD-parameters

Condition for Reciprocity

From Fig. 7.3(a), writing the $ABCD$-parameter equations,

$$\begin{aligned} V_s &= A \cdot 0 - B(-I_2') = BI_2' \\ I_1 &= C \cdot 0 - D(-I_2') = DI_2' \end{aligned} \quad \Rightarrow \quad \frac{I_2'}{V_s} = \frac{1}{B} \tag{7.7}$$

From Fig. 7.3(b), writing the $ABCD$-parameter equations,

$$\begin{aligned} 0 &= AV_s - BI_2 \\ -I_1' &= CV_s - DI_2 \end{aligned} \quad \Rightarrow \quad \frac{I_1'}{V_s} = \frac{AD - BC}{B} \tag{7.8}$$

From the principle of reciprocity, the condition for reciprocity is, $\frac{1}{B} = \frac{(AD - BC)}{B}$

$$\boxed{\mathbf{(AD - BC) = 1}}$$

Condition for Symmetry

From Eq. (7.7), $I_1 = DI_2' = D\frac{V_s}{B}$ (7.9)

From Eq. (7.8), $I_2 = \frac{I_1' + CV_s}{D} = \frac{1}{D}\left\{V_s\left(\frac{AD - BC}{B}\right) + CV_s\right\} = V_s\frac{A}{B}$ (7.10)

From Eq. (7.9) and Eq. (7.10), we have the condition for symmetry as,

$$\boxed{\mathbf{A = D}}$$

4. Conditions in terms of h-parameters

Condition for Reciprocity

From Fig. 7.3(a), writing the h-parameter equations,

$$\begin{aligned} V_s &= h_{11}I_1 + h_{12}\cdot 0 = h_{11}I_1 \\ -I_2' &= h_{21}I_1 + h_{22}\cdot 0 = h_{21}I_1 \end{aligned} \quad \Rightarrow \quad \frac{I_2'}{V_s} = -\frac{h_{21}}{h_{11}} \tag{7.11}$$

From Fig. 7.3(b), writing the h-parameter equations,

$$\begin{aligned} 0 &= -h_{11}I_1' + h_{12}V_s \\ I_2 &= -h_{21}I_1' + h_{22}V_s \end{aligned} \quad \Rightarrow \quad \frac{I_1'}{V_s} = \frac{h_{12}}{h_{11}} \tag{7.12}$$

From the principle of reciprocity, the condition for reciprocity is,

$$\boxed{\mathbf{h_{12} = -h_{21}}}$$

From Eq. (7.11), $I_1 = \dfrac{V_s}{h_{11}}$ (7.13)

From Eq. (7.12), $I_2 = -h_{21}\left(\dfrac{h_{12}}{h_{11}}V_s\right) + h_{22}V_s = V_s\dfrac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}$ (7.14)

From Eq. (7.13) and Eq. (7.14), we have the condition for symmetry as,

$$\boxed{(h_{11}h_{22} - h_{12}h_{21}) = 1}$$

Table 7.1 *Conditions of Reciprocity and Symmetry in terms of different Two-Port Parameters*

Parameter	*Condition of Reciprocity*	*Condition of Symmetry*
z	$z_{12} = z_{21}$	$z_{11} = z_{22}$
y	$y_{12} = y_{21}$	$y_{11} = y_{22}$
T ($ABCD$)	$(AD - BC) = 1$	$A = D$
h	$h_{12} = -h_{21}$	$(h_{11}h_{22} - h_{12}h_{21}) = 1$

7.4 INTERRELATIONSHIPS BETWEEN TWO-PORT PARAMETERS

Each type of two-port parameter has its own utility and is suited for certain specific applications. However, it is sometime necessary to convert one set of parameters to another. It is possible through simple mathematical manipulations to convert one set to any of the remaining sets. It is discussed below.

1. z-parameters in Terms of Other Parameters

(a) In terms of y-parameters

The z-parameter equations are,

$$V_1 = z_{11}I_1 + z_{12}I_2 \tag{7.15}$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

The y-parameter equations are,

$$I_1 = y_{11}V_1 + y_{12}V_2 \quad (7.16)$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

From Eq. (7.16), $V_2 = \frac{I_2}{y_{22}} - \frac{y_{21}}{y_{22}}V_1$; substituting this in first equation,

$$I_1 = y_{11}V_1 + y_{12}\left(\frac{I_2}{y_{22}} - \frac{y_{21}}{y_{22}}V_1\right) \quad \text{or} \quad V_1 = \frac{y_{22}}{\Delta y}I_1 - \frac{y_{12}}{\Delta y}I_2 \quad (7.17)$$

where, $\Delta y = (y_{11}y_{22} - y_{12}y_{21})$

Substituting this value in second equation of Eq. 7.16

$$I_2 = y_{21}\left(\frac{y_{22}}{\Delta y}I_1 - \frac{y_{12}}{\Delta y}I_2\right) + y_{22}V_2 \quad \text{or,} \quad V_2 = -\frac{y_{21}}{\Delta y}I_1 + \frac{y_{11}}{\Delta y}I_2 \quad (7.18)$$

Comparing Eqs (7.15), (7.17) and (7.18), we get,

$$\boxed{z_{11} = \frac{y_{22}}{\Delta y};\ z_{12} = -\frac{y_{12}}{\Delta y};\ z_{21} = -\frac{y_{21}}{\Delta y};\ z_{22} = \frac{y_{11}}{\Delta y}}$$

(b) In terms of transmission parameters

The Transmission parameter equations are,

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2 \quad (7.19)$$

From second equation of Eq. (7.19),

$$V_2 = \left(\frac{1}{C}\right)I_1 + \left(\frac{D}{C}\right)I_2 \quad (7.20)$$

From first equation of Eq. (7.19),

$$V_1 = A\left[\left(\frac{1}{C}\right)I_1 + \left(\frac{D}{C}\right)I_2\right] - BI_2 = \left(\frac{A}{C}\right)I_1 + \left(\frac{AD - BC}{C}\right)I_2 \quad (7.21)$$

Comparing Eq. (7.20) and (7.21) with Eq. (7.15), we get,

$$\boxed{z_{11} = \frac{A}{C};\ z_{12} = \frac{AD - BC}{C} = \frac{\Delta T}{C};\ z_{21} = \frac{1}{C};\ z_{22} = \frac{D}{C}}$$

(c) In terms of hybrid parameters

The hybrid parameter equations are,

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2 \quad (7.22)$$

From second equation, $V_2 = \left(-\frac{h_{21}}{h_{22}}\right)I_1 + \left(\frac{1}{h_{22}}\right)I_2 \quad (7.23)$

From first equation, $V_1 = h_{11}I_1 + h_{12}\left[\left(-\frac{h_{21}}{h_{22}}\right)I_1 + \left(\frac{1}{h_{22}}\right)I_2\right] = \left(\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}\right)I_1 + \left(\frac{h_{12}}{h_{22}}\right)I_2 \quad (7.24)$

Comparing Eqs (7.23) and (7.24) with Eq. (7.15), we get,

$$z_{11} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}};\ z_{12} = \frac{h_{12}}{h_{22}};\ z_{21} = -\frac{h_{21}}{h_{22}};\ z_{22} = \frac{1}{h_{22}}$$

Similarly, the inter-relation of the other parameter in terms of the remaining parameters is obtained by writing the remaining parameter equations in the same format as those of the other parameter; and comparing the co-efficients of the two sets of equations, a relation is obtained.

A summary of the relationships between impedance z-parameters, admittance y-parameters, hybrid h-parameters, and transmission $ABCD$-parameters is shown in Table where $\Delta z = (z_{11}z_{22} - z_{12}z_{21})$, $\Delta h = (h_{11}h_{22} - h_{12}h_{21})$, $\Delta T = (AD - BC)$, $\Delta T' = (A'D' - B'C')$, and $\Delta g = (g_{11}g_{22} - g_{12}g_{21})$.

Table 7.2 *Interrelationships between Two-Port Parameters*

	[z]	[y]	[ABCD]	[A′B′C′D′]	[h]	[g]
[z]	$\begin{matrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{matrix}$	$\begin{matrix} \frac{y_{22}}{\Delta y} & -\frac{y_{12}}{\Delta y} \\ \frac{-y_{21}}{\Delta y} & \frac{y_{11}}{\Delta y} \end{matrix}$	$\begin{matrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{matrix}$	$\begin{matrix} \frac{D'}{C'} & \frac{1}{C'} \\ \frac{\Delta T'}{C'} & \frac{A'}{C'} \end{matrix}$	$\begin{matrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{g_{11}} & -\frac{g_{12}}{g_{11}} \\ \frac{g_{21}}{h_{11}} & \frac{\Delta g}{g_{11}} \end{matrix}$
[y]	$\begin{matrix} \frac{z_{22}}{\Delta z} & -\frac{z_{12}}{\Delta z} \\ -\frac{z_{21}}{\Delta z} & \frac{z_{11}}{\Delta z} \end{matrix}$	$\begin{matrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{matrix}$	$\begin{matrix} \frac{D}{B} & -\frac{\Delta T}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{matrix}$	$\begin{matrix} \frac{A'}{B'} & -\frac{1}{B'} \\ -\frac{\Delta T'}{B'} & \frac{D'}{B'} \end{matrix}$	$\begin{matrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{matrix}$	$\begin{matrix} \frac{\Delta g}{g_{22}} & \frac{g_{12}}{g_{22}} \\ -\frac{g_{21}}{g_{22}} & \frac{1}{g_{22}} \end{matrix}$
[ABCD]	$\begin{matrix} \frac{z_{11}}{z_{21}} & \frac{\Delta z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{matrix}$	$\begin{matrix} -\frac{y_{22}}{y_{21}} & -\frac{1}{y_{21}} \\ -\frac{\Delta y}{y_{21}} & -\frac{y_{11}}{y_{21}} \end{matrix}$	$\begin{matrix} A & B \\ C & D \end{matrix}$	$\begin{matrix} \frac{D'}{\Delta T'} & \frac{B'}{\Delta T'} \\ \frac{C'}{\Delta T'} & \frac{A'}{\Delta T'} \end{matrix}$	$\begin{matrix} -\frac{\Delta h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{matrix}$	$\begin{matrix} \frac{1}{g_{21}} & -\frac{g_{22}}{g_{21}} \\ \frac{g_{11}}{g_{21}} & \frac{\Delta g}{g_{21}} \end{matrix}$
[A′B′C′D′]	$\begin{matrix} \frac{z_{22}}{z_{12}} & \frac{\Delta z}{z_{12}} \\ \frac{1}{z_{12}} & \frac{z_{11}}{z_{12}} \end{matrix}$	$\begin{matrix} -\frac{y_{11}}{y_{12}} & -\frac{1}{y_{12}} \\ -\frac{\Delta y}{y_{12}} & -\frac{y_{22}}{y_{12}} \end{matrix}$	$\begin{matrix} \frac{D}{\Delta T} & \frac{B}{\Delta T} \\ \frac{C}{\Delta T} & \frac{A}{\Delta T} \end{matrix}$	$\begin{matrix} A' & B' \\ C' & D' \end{matrix}$	$\begin{matrix} \frac{1}{h_{22}} & \frac{h_{11}}{h_{12}} \\ \frac{h_{22}}{h_{12}} & \frac{\Delta h}{h_{12}} \end{matrix}$	$\begin{matrix} -\frac{\Delta g}{g_{12}} & -\frac{g_{22}}{g_{12}} \\ -\frac{g_{11}}{g_{12}} & -\frac{1}{g_{12}} \end{matrix}$
[h]	$\begin{matrix} \frac{\Delta z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ -\frac{z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{y_{11}} & -\frac{y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta y}{y_{11}} \end{matrix}$	$\begin{matrix} \frac{B}{D} & \frac{\Delta T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{matrix}$	$\begin{matrix} \frac{B'}{A'} & \frac{1}{A'} \\ -\frac{\Delta T'}{A'} & \frac{D'}{B'} \end{matrix}$	$\begin{matrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{matrix}$	$\begin{matrix} \frac{g_{22}}{\Delta g} & -\frac{g_{12}}{\Delta g} \\ -\frac{g_{211}}{\Delta g} & \frac{g_{11}}{\Delta g} \end{matrix}$
[g]	$\begin{matrix} \frac{1}{z_{11}} & -\frac{z_{12}}{z_{11}} \\ \frac{z_{21}}{z_{11}} & \frac{\Delta z}{z_{11}} \end{matrix}$	$\begin{matrix} \frac{\Delta y}{y_{22}} & \frac{y_{12}}{y_{22}} \\ -\frac{y_{21}}{y_{22}} & -\frac{1}{y_{22}} \end{matrix}$	$\begin{matrix} \frac{C}{A} & -\frac{\Delta T}{A} \\ \frac{1}{A} & \frac{B}{A} \end{matrix}$	$\begin{matrix} \frac{C'}{D'} & -\frac{1}{D'} \\ \frac{\Delta T'}{D'} & \frac{B'}{D'} \end{matrix}$	$\begin{matrix} \frac{h_{22}}{\Delta h} & -\frac{h_{12}}{\Delta h} \\ -\frac{h_{21}}{\Delta h} & \frac{h_{11}}{\Delta h} \end{matrix}$	$\begin{matrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{matrix}$

7.5 INTERCONNECTION OF TWO-PORT NETWORKS

In certain applications, it becomes necessary to connect the two-port networks together.
The common connections are (a) series, (b) parallel and (c) cascade.

(a) Series Connection of Two-port Networks

As in the case of elements, a series connection is defined when the currents in the series elements are equal and the voltages add up to give the resultant voltage.

In the case of two-port networks, this property must be applied individually to each of the ports. Thus, if we consider 2 networks ***r*** and ***s*** connected in series

At port 1,

$$I_{r1} = I_{s1} = I_1, \text{ and } V_{r1} + V_{s1} = V_1$$

Similarly, at port 2,

$$I_{r2} = I_{s2} = I_2 \text{ and } V_{r2} + V_{s2} = V_2$$

The two networks, ***r*** and ***s*** can be connected in the following manner to be in series with each other.

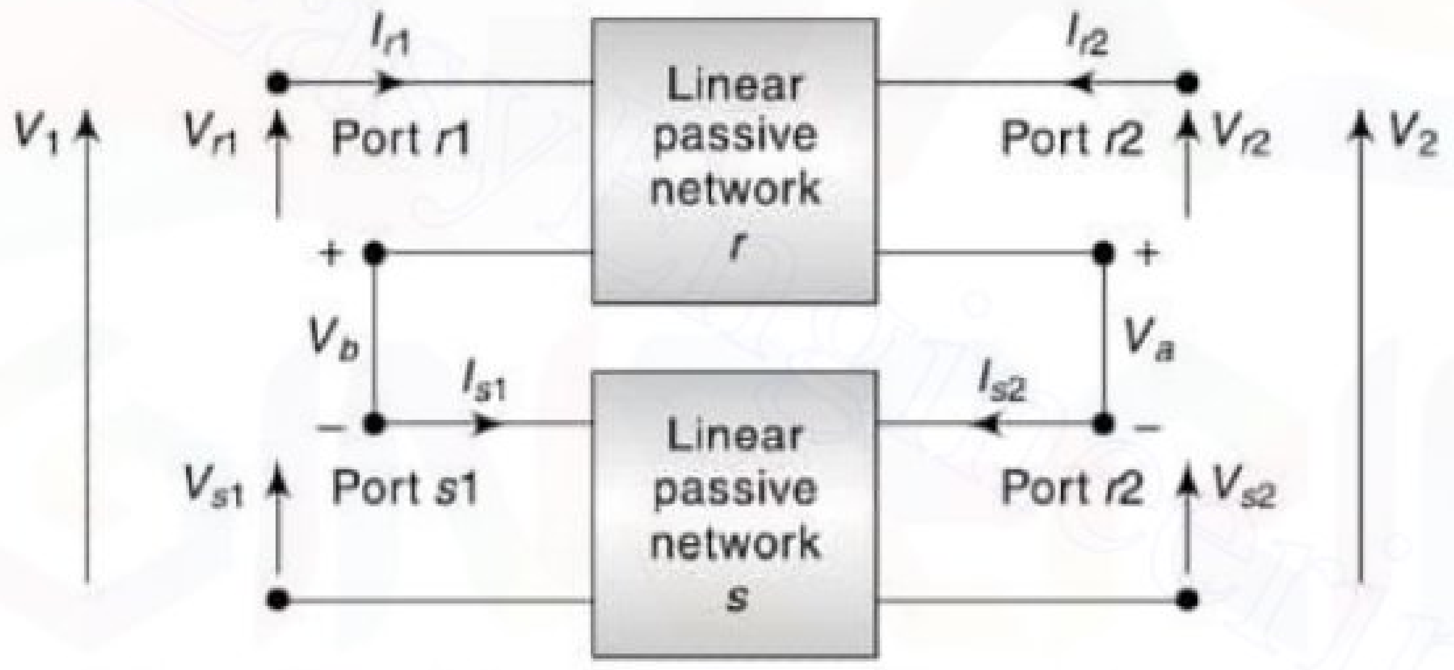


Figure 7.4 *Series connection of two-port networks*

Under these conditions,

$$V_1 = (V_{r1} + V_{s1}) = (z_{11r} + z_{11s})I_1 + (z_{12r} + z_{12s})I_2$$

$$V_2 = (V_{r2} + V_{s2}) = (z_{21r} + z_{21s})I_1 + (z_{22r} + z_{22s})I_2$$

It is seen that, the ***resultant impedance parameter matrix for the series combination is the addition of the two individual impedance matrices.***

$$[Z] = [Zr] + [Zs]$$

Note: In the interconnection of series networks, there is a strong requirement of isolation, since the ground node of upper network form the non-ground node of the lower network. For the port properties to be valid, the voltages V_a and V_b must be identically zero for the two networks r and s to be connected in series. If V_a and V_b are not zero, then by connecting the two ports there will be a circulating current and port property of the individual networks r and s will be violated.

(b) Parallel Connection of Two-port Networks

As in the case of elements, a parallel connection is defined when the voltages in the parallel elements are equal and the currents add up to give the resultant current.

In the case of two-port networks, this property must be applied individually to each of the ports. Thus, if we consider 2 networks ***r*** and ***s*** connected in parallel,

At port 1,

$$I_{r1} + I_{s1} = I_1, \text{ and } V_{r1} = V_{s1} = V_1$$

Similarly, at port 2,

$$I_{r2} + I_{s2} = I_2 \text{ and } V_{r1} = V_{s1} = V_1$$

The two networks, ***r*** and ***s*** can be connected in following manner to be in parallel with each other. Under these conditions,

$$I_1 = (I_{r1} + I_{s1}) = (y_{11r} + y_{11s})V_1 + (y_{12r} + y_{12s})V_2$$

$$I_2 = (I_{2r} + I_{2s}) = (y_{21r} + y_{21s})V_1 + (y_{22r} + y_{22s})V_2$$

It is seen that, the *resultant admittance parameter matrix for the parallel combination is the addition of the two individual admittance matrices.*

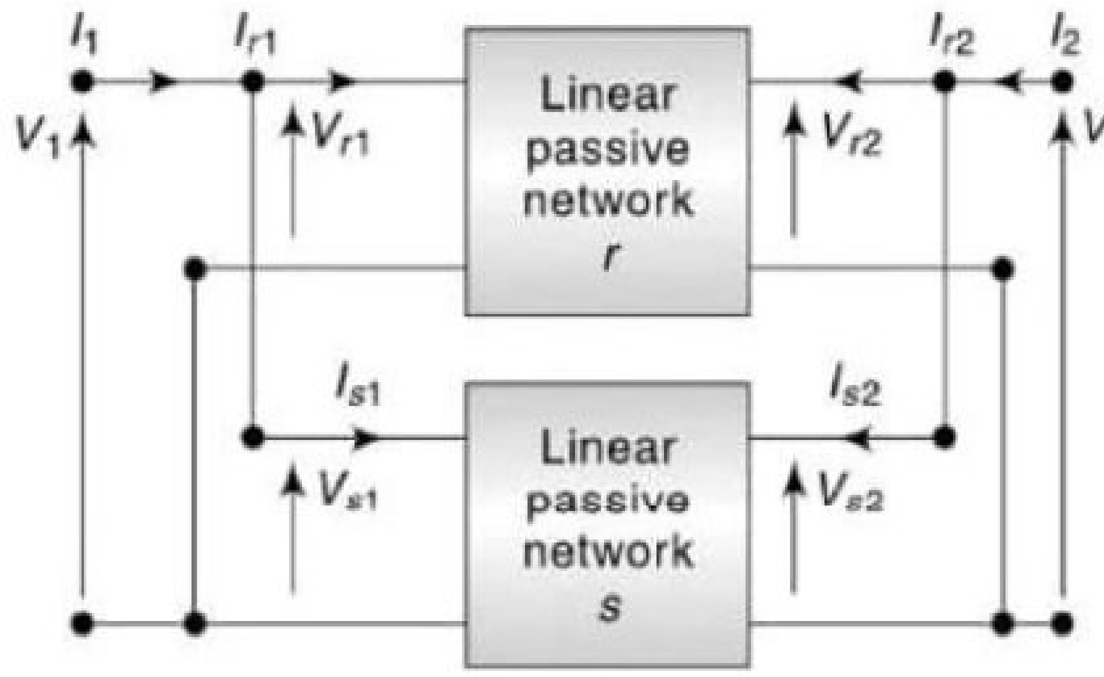


Figure 7.5 *Parallel connection of two-port networks*

$$[Y] = [Yr] + [Ys]$$

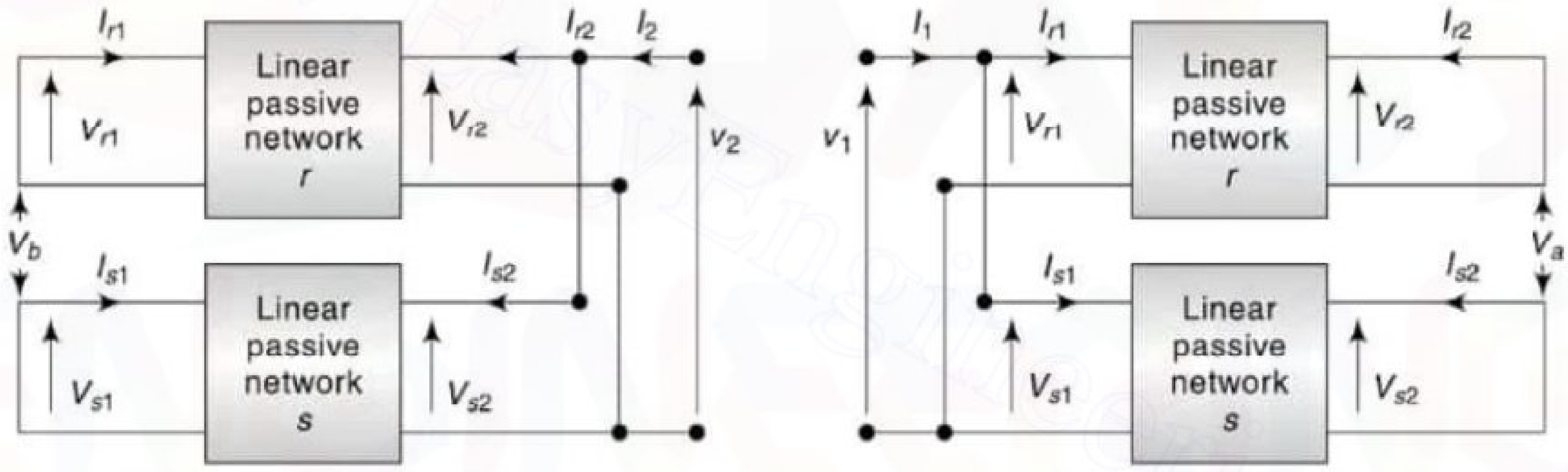


Figure 7.6(a) $V_b = 0$ **Figure 7.6(b)** $V_a = 0$

Note: As in series connection, parallel connection is also possible under the condition that $V_a = V_b = 0$; otherwise they cannot be connected in parallel as that will violate the port properties.

(c) Cascade Connection of Two-port Networks

A cascade connection is defined when the output of one network becomes the input to the next network.

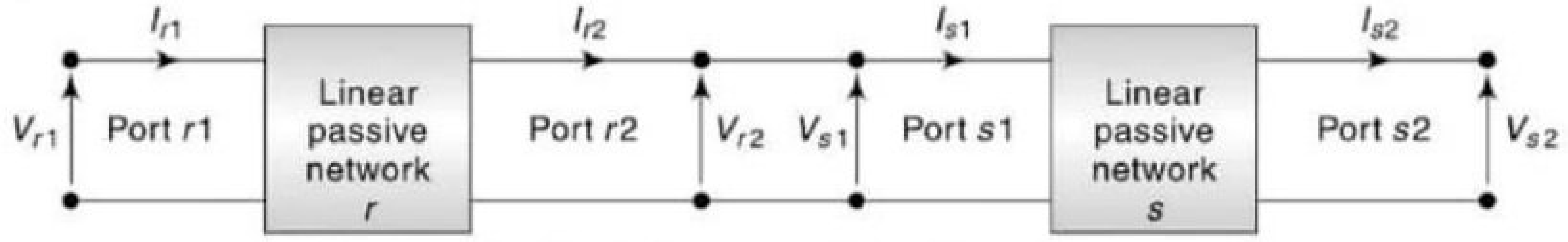


Figure 7.7 *Cascade connection of two-port network*

It can be easily seen that $I_{r2} = I_{s1}$ and $V_{r2} = V_{s1}$.

Therefore it can easily be seen that the *ABCD* parameters are the most suitable to be used for this connection.

$$\begin{bmatrix} V_{r1} \\ I_{r1} \end{bmatrix} = \begin{bmatrix} A_r & B_r \\ C_r & D_r \end{bmatrix} \begin{bmatrix} V_{r2} \\ I_{r2} \end{bmatrix}, \quad \begin{bmatrix} V_{s1} \\ I_{s1} \end{bmatrix} = \begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} \begin{bmatrix} V_{s2} \\ I_{s2} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{r1} \\ I_{r1} \end{bmatrix} = \begin{bmatrix} A_r & B_r \\ C_r & D_r \end{bmatrix} \begin{bmatrix} V_{r2} \\ I_{r2} \end{bmatrix} = \begin{bmatrix} A_r & B_r \\ C_r & D_r \end{bmatrix} \begin{bmatrix} V_{s1} \\ I_{s1} \end{bmatrix} = \begin{bmatrix} A_r & B_r \\ C_r & D_r \end{bmatrix} \begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} \begin{bmatrix} V_{s2} \\ I_{s2} \end{bmatrix}$$

$$= \begin{bmatrix} A_r & B_r \\ C_r & D_r \end{bmatrix} \begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Thus it is seen that the *overall ABCD matrix is the product of the two individual ABCD matrices.* This is a very useful property in practice, especially when analyzing transmission lines.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_r & B_r \\ C_r & D_r \end{bmatrix} \begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix}$$

7.6 TWO-PORT NETWORK FUNCTIONS

Two-port network functions are broadly divided into two groups:

1. Transfer function, and
2. Driving point functions.

7.6.1 Transfer Function

It is defined as the ratio of an output transform to an input transform, with zero initial condition and with no internal energy sources ecxcept the controlled sources.

For a two-port network, having the variables $I_1(s)$, $I_2(s)$, $V_1(s)$ and $V_2(s)$, the transfer function can take the following four forms:

Voltage Transfer Function $G_{12}(s) = \dfrac{V_1(s)}{V_2(s)}; G_{21}(s) = \dfrac{V_2(s)}{V_1(s)}$

Current Transfer Function $\alpha_{12}(s) = \dfrac{I_1(s)}{I_2(s)}; \alpha_{21}(s) = \dfrac{I_2(s)}{I_1(s)}$

Transfer Impedance Function $Z_{12}(s) = \dfrac{V_1(s)}{V_2(s)}; Z_{21}(s) = \dfrac{V_2(s)}{V_1(s)}$

Transfer Admittance Function $Y_{12}(s) = \dfrac{I_1(s)}{I_2(s)}; Y_{21}(s) = \dfrac{I_2(s)}{I_1(s)}$

Note: *(i) For a one-port network, Z(s) = 1/Y(s); but for a two-port network, in general* $Z_{12} \neq 1/Y_{12}$; $G_{12} \neq 1/\alpha_{12}$;

(ii) Z and Y functions will becomes z and y parameters under the conditions of open-circuits or short-circuits, respectively.

7.6.2 Driving Point Function

It takes two forms:

Driving Point Impedance [Z(s)] For a two-port newtork in zero state with no internal energy sourceds, the driving point impedance s the ratio of transform voltage at any port to the transform current at the same port.

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)};\ Z_{22}(s) = \frac{V_2(s)}{I_2(s)}$$

Driving Point Admittance [Y(s)] For a two-port network in zero state with no internal energy sources, the driving point admittance is the ratio of transform current at any port to the transform voltage at the same port

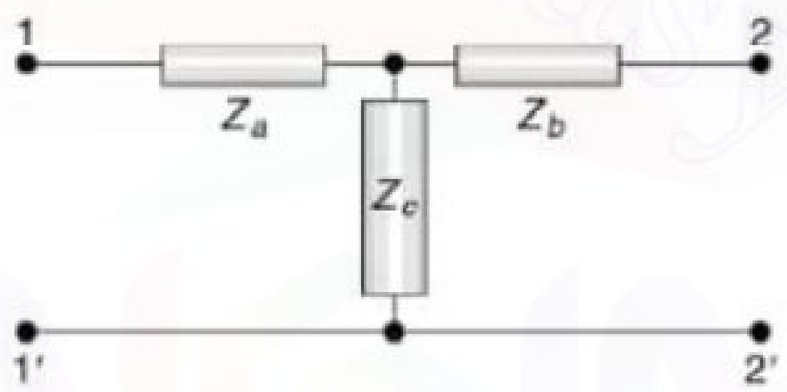
$$Y_{11}(s) = \frac{I_1(s)}{V_1(s)};\ Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$$

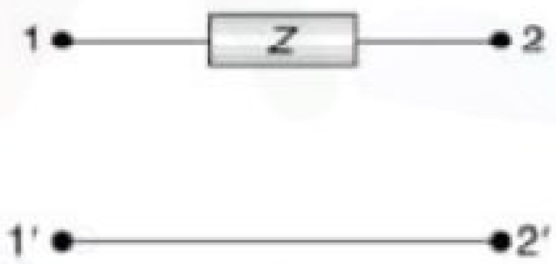
Note: *(i) Driving point impedance and admittance functions together are known as immittance function.*

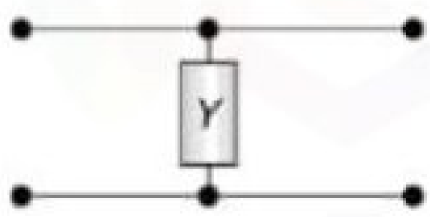
(2) Z and Y functions will becomes z and y parameters under the conditions of open circuits or short circuits,

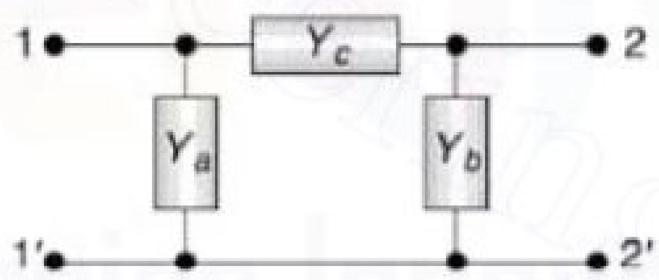
SOLVED PROBLEMS

7.1 Find the Z and Y parameter for the networks shown in figure.

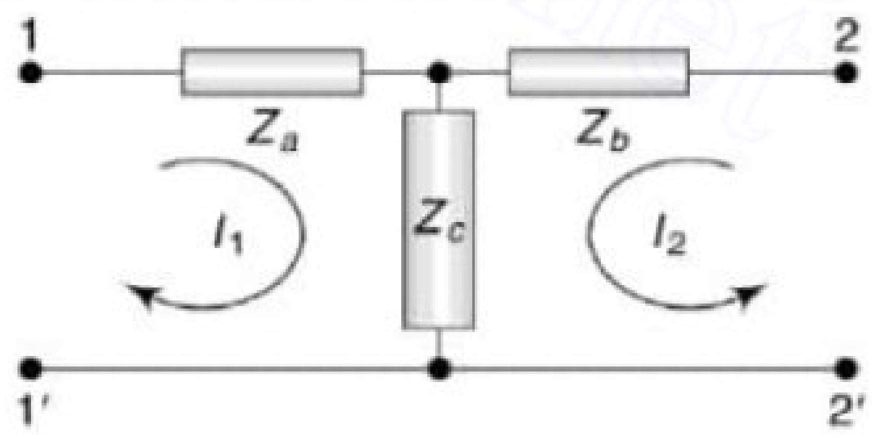
(a) 

(b) 

(c) 

(d) 

Solution



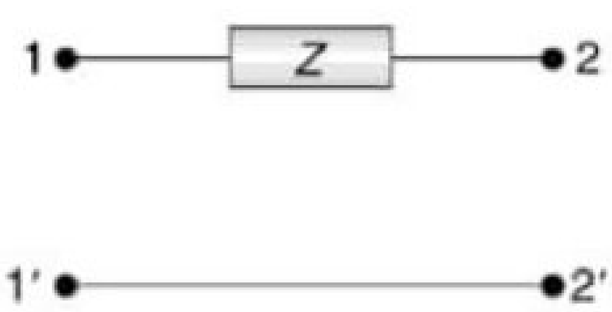
(a) By KVL, $(Z_a + Z_c)I_1 + Z_cI_2 = V_1$

and $Z_cI_1 + (Z_b + Z_c)I_2 = V_2$

Thus, the Z-parameters are:

$$z_{11} = (Z_a + Z_c),\ z_{12} = z_{21} = Z_c,\quad z_{22} = (Z_b + Z_c)$$

(b) By KCL,



$$I_1 = \frac{V_1 - V_2}{Z} = \frac{1}{Z}V_1 - \frac{1}{Z}V_2$$

and $$I_2 = \frac{V_2 - V_1}{Z} = -\frac{1}{Z}V_1 + \frac{1}{Z}V_2$$

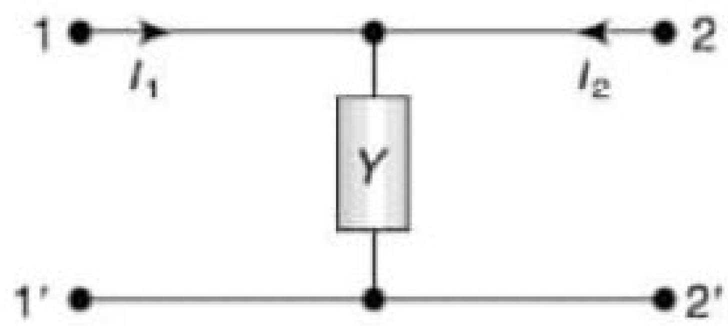
Thus, the y-parameters are,

$$y_{11} = \frac{1}{Z} = y_{22}\quad y_{12} = y_{21} = -\frac{1}{Z}$$

Since, $\Delta y = y_{11}y_{22} - y_{12}y_{21} = 0$, the z-parameters do not exist for this network.

(c) By KVL,

$$V_1 = \frac{I_1 + I_2}{Y} = V_2 \quad \text{or, } V_1 = \left(\frac{1}{Y}\right)I_1 + \left(\frac{1}{Y}\right)I_2 \quad \text{and} \quad V_2 = \left(\frac{1}{Y}\right)I_1 + \left(\frac{1}{Y}\right)I_2$$

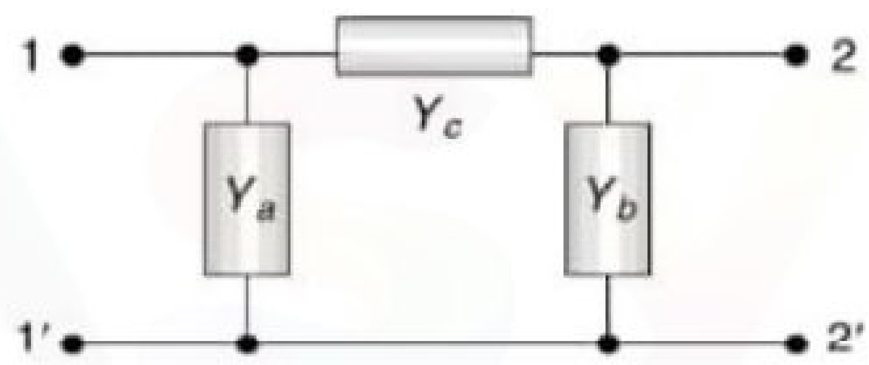


Thus, the z-parameters are,

$$z_{11} = z_{22} = \frac{1}{Y} = z_{12} = z_{21}$$

Since, $\Delta z = z_{11}z_{22} - z_{12}z_{21} = 0$, the y-parameters do not exist for this network.

(d) By KCL,



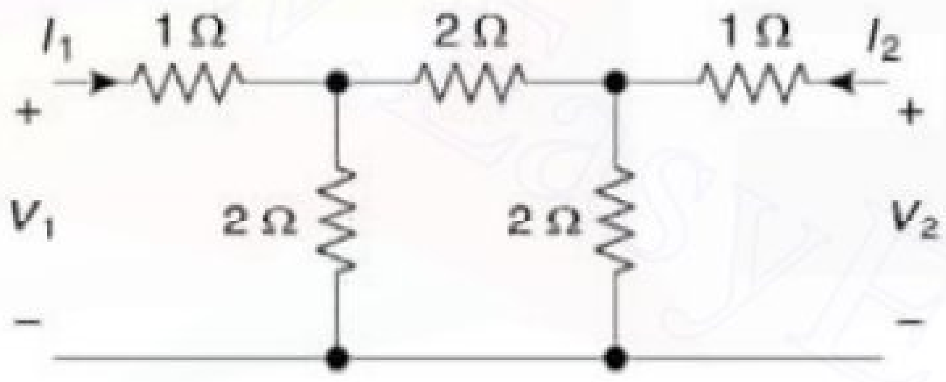
$$I_1 = Y_a V_1 + (V_1 - V_2)Y_c = V_1(Y_a + Y_c) - V_2 Y_c$$
$$I_2 = Y_b V_2 + (V_2 - V_1)Y_c = -V_1 Y_c + V_2(Y_b + Y_c)$$

Thus, the y-parameters are:

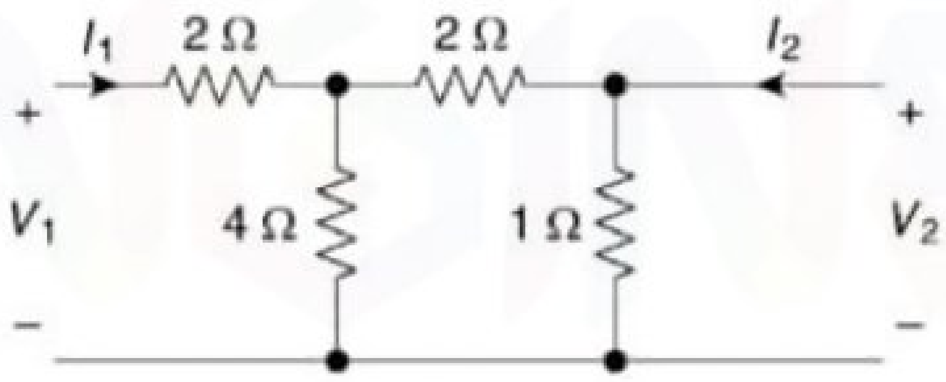
$$y_{11} = Y_a + Y_c;\ y_{12} = y_{21} = -Y_c;\ y_{22} = Y_b + Y_c$$

7.2 Obtain the z-parameters for the circuit shown in figure.

(a)

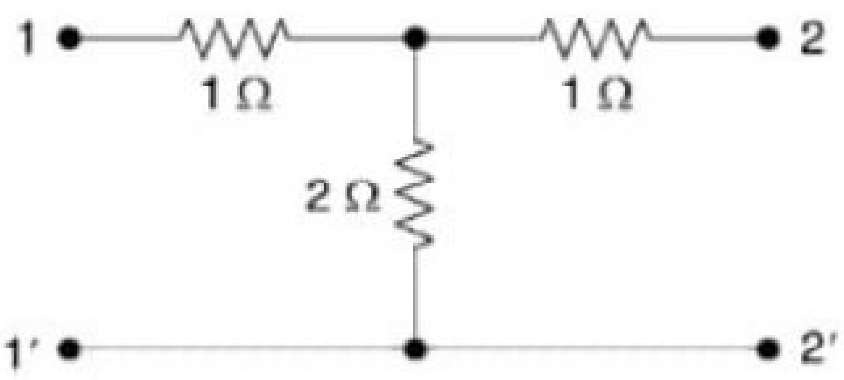


(b)

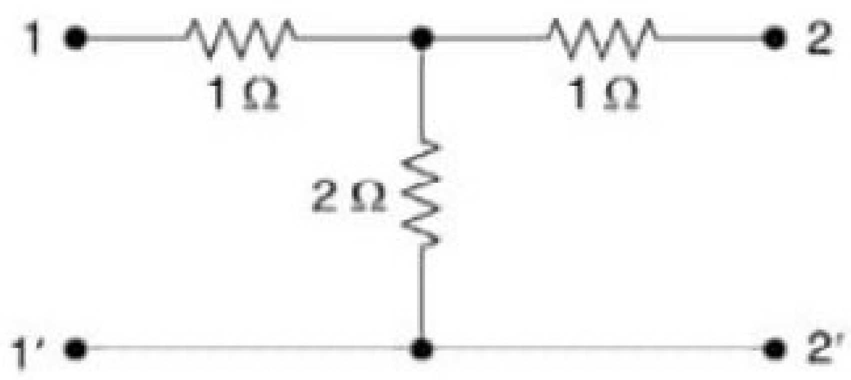


Solution

(a) The given circuit can be considered as the cascade connection of the following two networks:



(a)



(b)

From Prob. 7.1(a), $z_{11a} = z_{11b} = z_{22a} = z_{22b} = 3\Omega$

$$z_{12a} = z_{21a} = z_{12b} = z_{21b} = 2\Omega$$

So, the transmission parameters are,

$$\therefore \qquad A_a = A_b = \frac{z_{11}}{z_{21}} = \frac{3}{2}$$

$$\therefore \quad B_a = B_b = \frac{\Delta z}{z_{21}} = \frac{9-4}{2} = \frac{5}{2}\ \Omega$$

$$\therefore \quad C_a = C_b = \frac{1}{z_{21}} = \frac{1}{2}\ \mho$$

$$\therefore \quad D_a = D_b = \frac{z_{22}}{z_{21}} = \frac{3}{2}$$

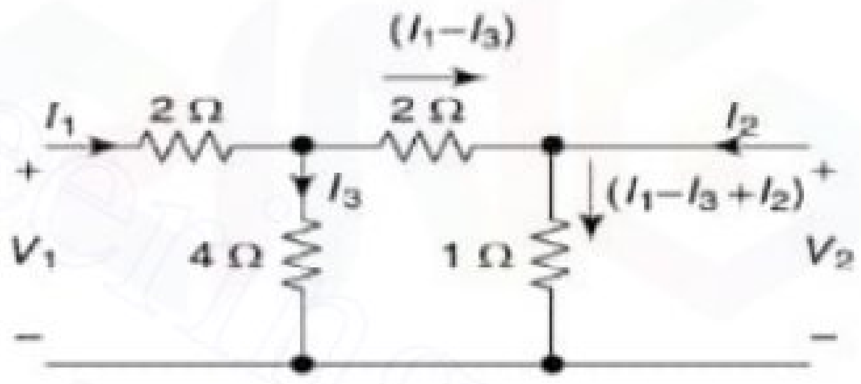
So, the transmission parameters of the resulting network are:

$$T = T_a \times T_b = \begin{bmatrix} 3/2 & 5/2 \\ 1/2 & 3/2 \end{bmatrix}\begin{bmatrix} 3/2 & 5/2 \\ 1/2 & 3/2 \end{bmatrix} = \begin{bmatrix} 7/2 & 15/2 \\ 3/2 & 7/2 \end{bmatrix}$$

So, the z-parameters are:

$$\left.\begin{aligned} z_{11} &= \frac{A}{C} = \frac{7}{3}\ \Omega \\ z_{12} &= \frac{\Delta T}{C} = \frac{2}{3}\ \Omega \\ z_{21} &= \frac{1}{C} = \frac{2}{3}\ \Omega \\ z_{22} &= \frac{D}{C} = \frac{7}{3}\ \Omega \end{aligned}\right\}$$

(b) By KVL,



$$V_1 = 2I_1 + 4I_3$$

$$V_2 = I_1 + I_2 - I_3$$

and $\quad 2(I_1 - I_3) + I_1 + I_2 - I_3 - 4I_3 = 0$

Eliminating I_3 from above equations,

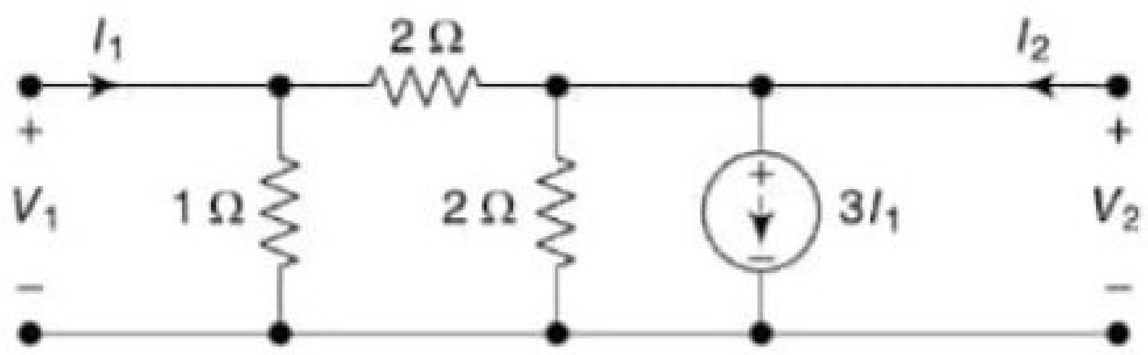
$$V_1 = \frac{26}{7}I_1 + \frac{4}{7}I_2$$

$$V_2 = \frac{4}{7}I_1 + \frac{6}{7}I_2$$

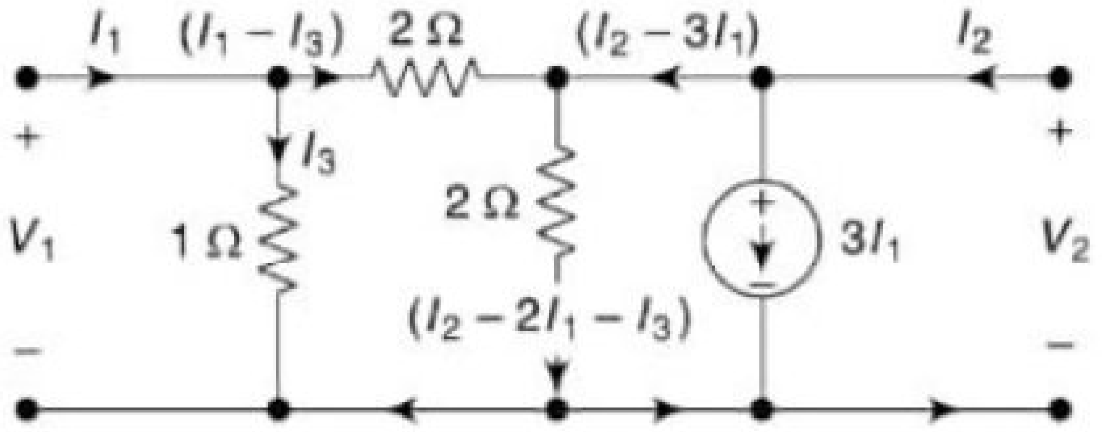
Thus, the z-parameters are:

$$[z] = \begin{bmatrix} 26/7 & 4/7 \\ 4/7 & 6/7 \end{bmatrix}\Omega$$

7.3 For the network shown, find z and y-parameters.



Solution From the figure, we can write the KVL equations,



$$V_1 = I_3 \qquad \text{(i)}$$

$$V_2 = 2I_2 - 4I_1 - 2I_3 \qquad \text{(ii)}$$

and, $$2I_1 - 2I_3 + 2I_2 - 4I_1 - 2I_3 - I_3 = 0 \quad \Rightarrow \quad I_3 = \frac{2}{5}(I_2 - I_1)$$

From (i), $V_1 = -\frac{2}{5}I_1 + \frac{2}{5}I_2 = -0.4I_1 + 0.4I_2$

From (ii), $V_2 = 2I_2 - 4I_1 - \frac{4}{5}I_2 + \frac{4}{5}I_1 = -3.2I_1 + 1.2I_2$

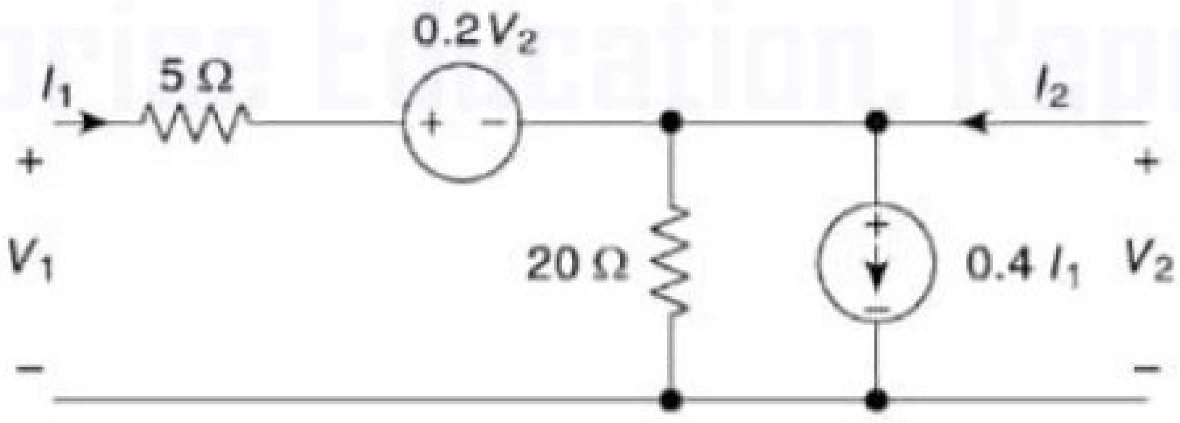
$$\therefore \qquad [z] = \begin{bmatrix} -0.4 & 0.4 \\ -3.2 & 1.2 \end{bmatrix} \Omega$$

$$\Delta z = (-0.4 \times 1.2) - 0(0.4) \times (-3.2) = 0.8$$

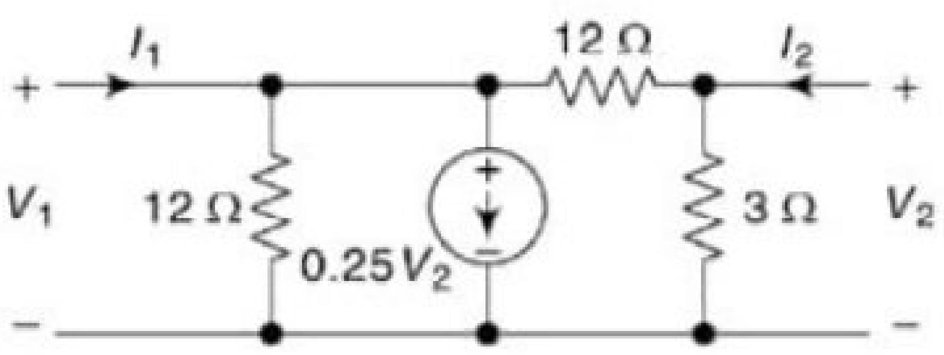
$$\therefore \qquad [y] = \begin{bmatrix} 1.2/0.8 & -\frac{0.4}{0.8} \\ 3.2/0.8 & -\frac{0.4}{0.8} \end{bmatrix} \mho = \begin{bmatrix} 1.5 & -0.5 \\ 4 & -0.5 \end{bmatrix} \mho$$

7.4 Find the y-parameters for the 2-port networks shown.

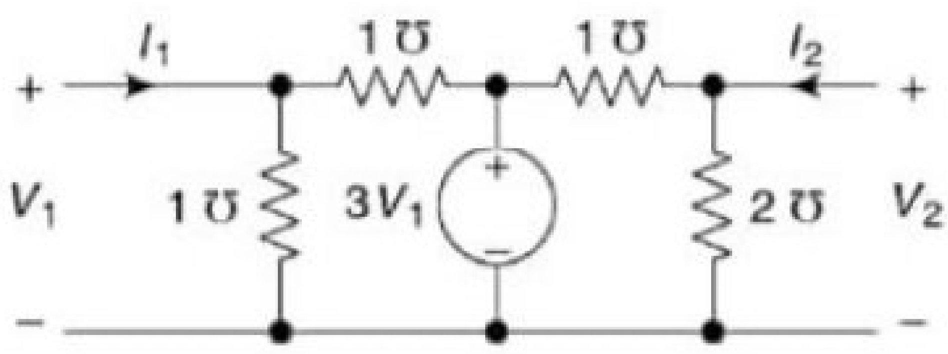
(a)



(b)



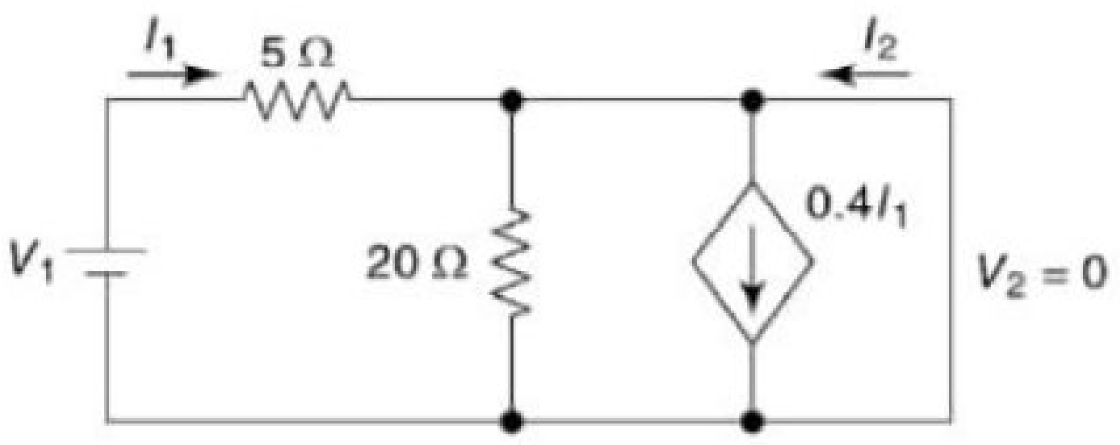
(c)

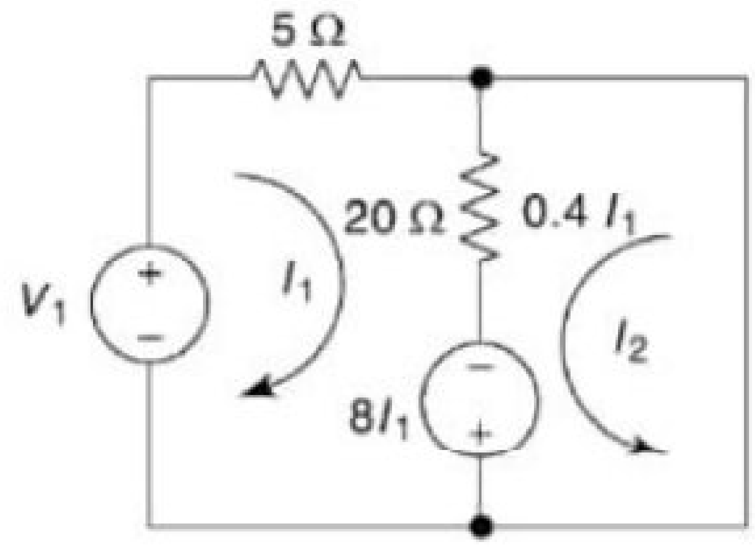


Solution

(a) We consider two cases to find out the y-parameters.

Case (I) Making port- 2 shorted and applying a voltage of V_1 at port- 1





By KVL,

$$17I_1 + 20I_2 = V_1$$

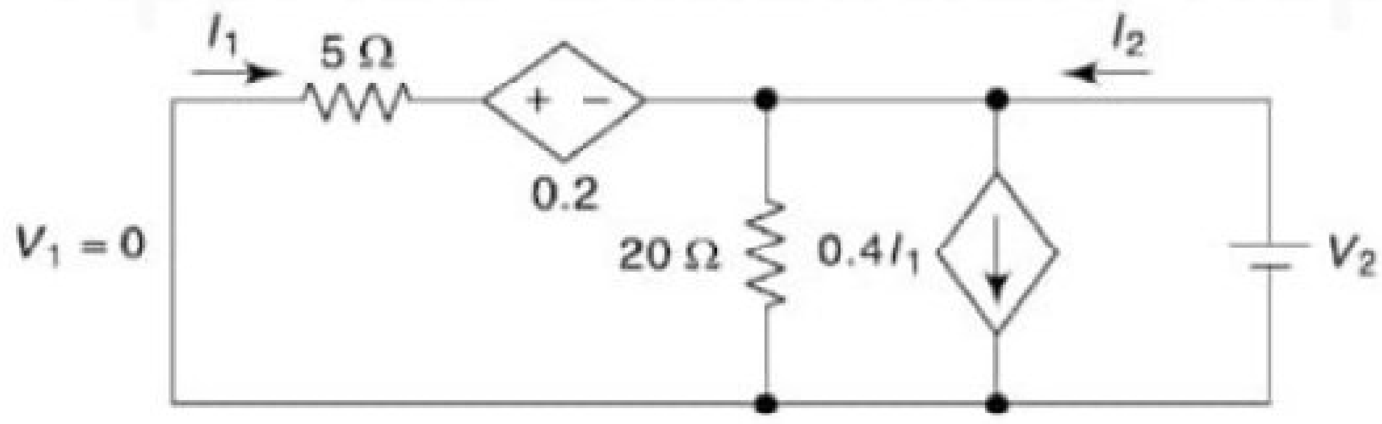
and $$12I_1 + 20I_2 = 0$$

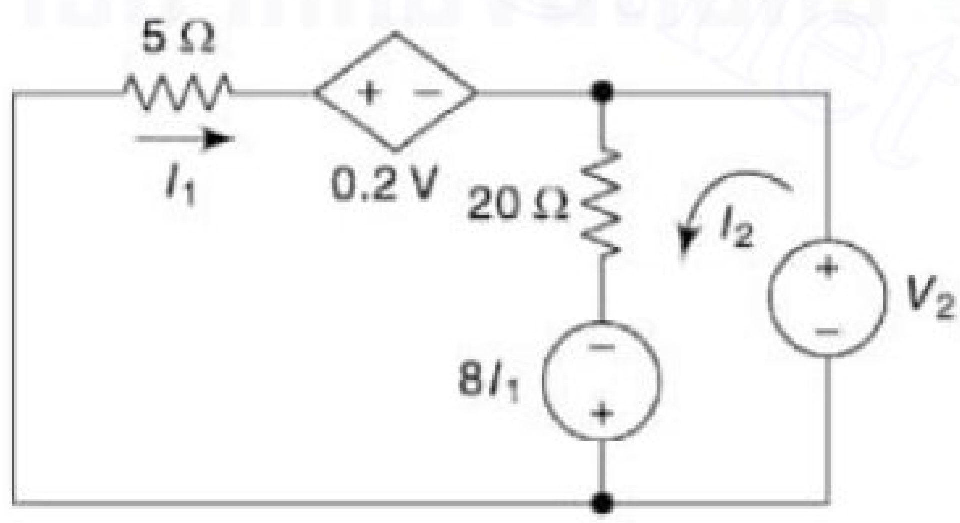
$$I_1 = \frac{\begin{vmatrix} V_1 & 20 \\ 0 & 20 \end{vmatrix}}{\begin{vmatrix} 17 & 20 \\ 12 & 20 \end{vmatrix}} = 0.2V_1 \Rightarrow y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 0.2\ \mho$$

Solving,

$$I_2 = \frac{\begin{vmatrix} 17 & V_1 \\ 12 & 0 \end{vmatrix}}{\begin{vmatrix} 17 & 20 \\ 12 & 20 \end{vmatrix}} = -0.12V_1 \Rightarrow y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -0.12\ \mho$$

Case (II) Making port-1 shorted and applying a voltage of V_2 at port- 2





By KVL,

$$17I_1 + 20I_2 = -0.2V_2$$

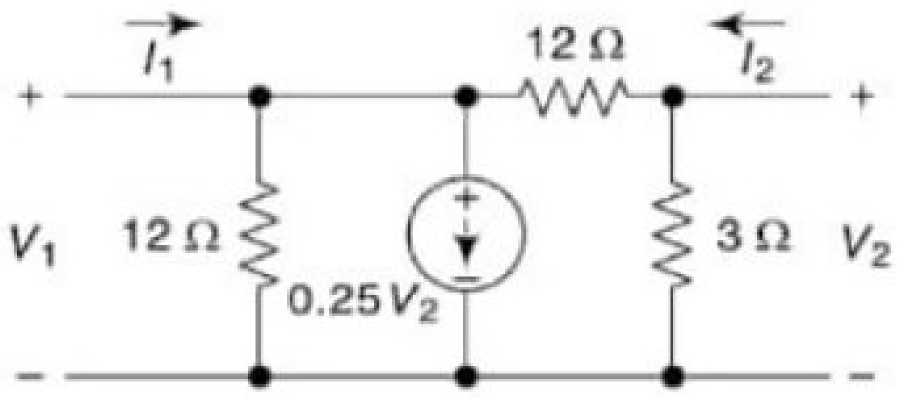
and $$12I_1 + 20I_2 = V_2$$

$$I_1 = \frac{\begin{vmatrix} -0.2V_2 & 20 \\ V_2 & 20 \end{vmatrix}}{\begin{vmatrix} 17 & 20 \\ 12 & 20 \end{vmatrix}} = -0.24V_2 \Rightarrow y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -0.24\ \mho$$

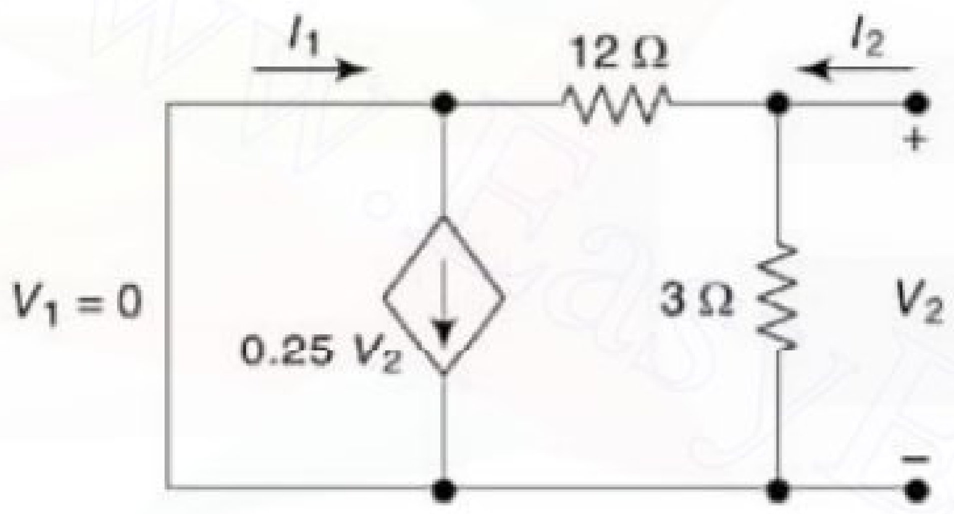
Solving,

$$I_2 = \frac{\begin{vmatrix} 17 & -0.2V_2 \\ 12 & V_2 \end{vmatrix}}{\begin{vmatrix} 17 & 20 \\ 12 & 20 \end{vmatrix}} = 0.194V_2 \Rightarrow y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = 0.194\ \mho$$

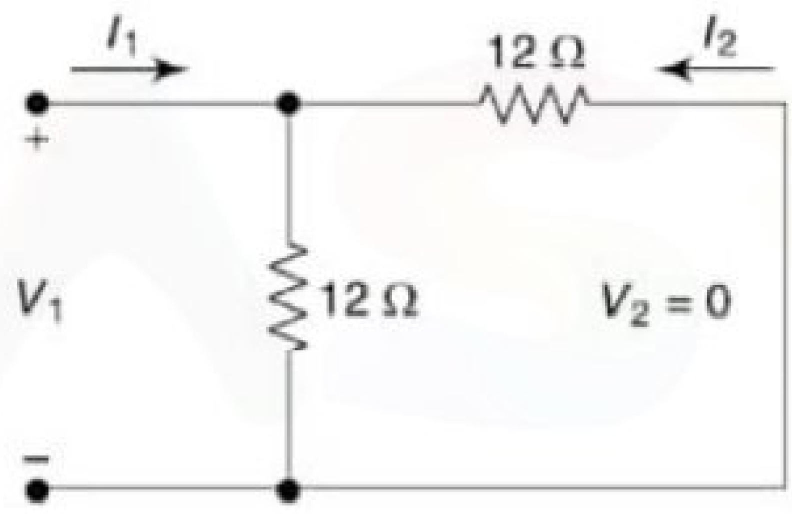
Thus, $[y] = \begin{bmatrix} 0.2 & -0.24 \\ -0.12 & 0.194 \end{bmatrix} \mho$



(b) We consider two cases.
Case (I) $V_1 = 0$
Case (II) $V_2 = 0$



Case (I): $V_1 = 0$



Case (II): $V_2 = 0$

By KCL,

$$I_1 = y_{12}V_2|_{V_1=0} = \frac{V_2}{4} + \left(\frac{0 - V_2}{12}\right) \Rightarrow y_{12} = \frac{1}{6}\ \mho$$

$$I_2 = y_{22}V_2|_{V_1=0} = \frac{V_2}{3} + \frac{V_2}{12} \Rightarrow y_{22} = \frac{5}{12}\ \mho$$

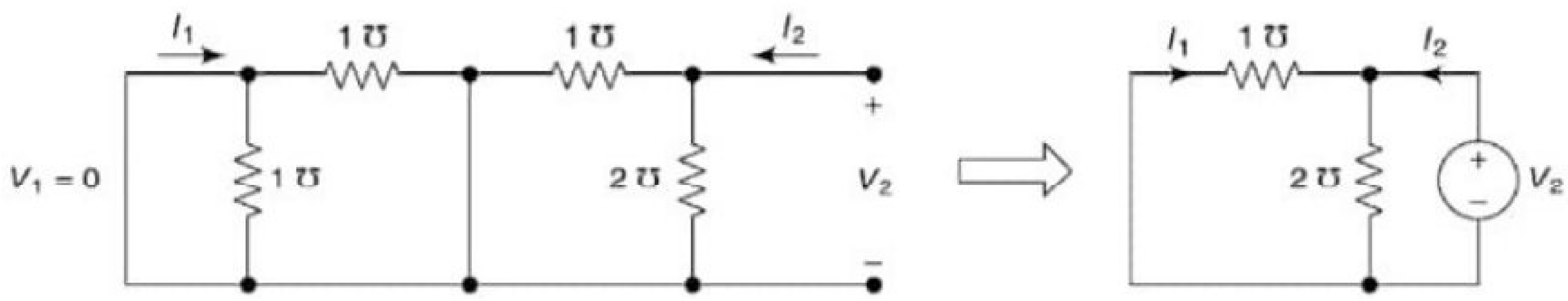
$$I_1 = y_{11}V_1|_{V_2=0} = \left(\frac{1}{12} + \frac{1}{12}\right)V_1 \Rightarrow y_{11} = \frac{1}{6}\ \mho$$

$$I_2 = y_{21}V_1|_{V_2=0} = -\frac{V_1}{12} \Rightarrow y_{21} = -\frac{1}{12}\ \mho$$

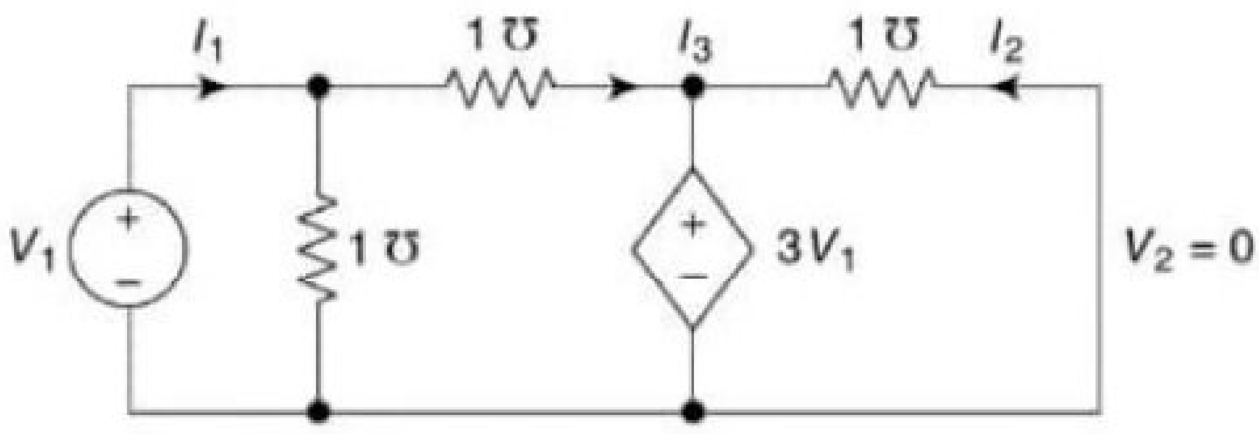
(c) For $V_1 = 0$, the circuit becomes as shown.

$\therefore$ $I_2 = y_{22}V_2 = (1 + 2)V_2 = 3V_2 \Rightarrow y_{22} = 3\ \mho$

Also, $-\frac{I_1}{1} = V_2 \Rightarrow y_{12} = -1\ \mho$



For $V_2 = 0$, the circuit becomes as shown.



$$\therefore \quad -\frac{I_2}{1} = 3V_1 \qquad \text{(i)}$$

$$\frac{I_3}{1} + 3V_1 = V_1 \Rightarrow 2V_1 = -I_3 \qquad \text{(ii)}$$

$$I_1 = I_3 + I_4 \qquad \text{(iii)}$$

$$\text{and} \quad V_1 = \frac{I_4}{1} \qquad \text{(iv)}$$

From (i) to (iv),

$$I_1 = V_1 + I_3 = V_1 - 2V_1 = -V_1 \quad \Rightarrow \quad y_{11} = -1\ ℧$$

From (i), $y_{21} = -3$ ℧

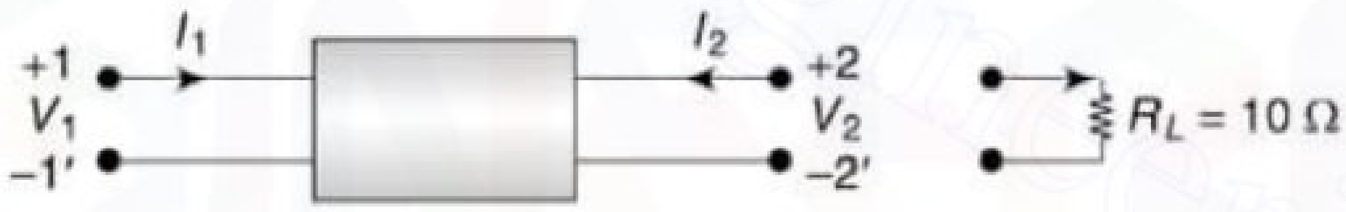
Thus, the y-parameters are:

$$[y] = \begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix} ℧$$

From the interrelationship, we get the z-parameters as:

$$[z] = \begin{bmatrix} -1 & 0 \\ -1 & 1/3 \end{bmatrix} (\Omega)$$

7.5 Measurements were made on a two-port network shown in the figure.



(i) With port-2 open, a voltage of $100\angle 0°$ volt is applied to port-1, resulted in, $I_1 = 10\angle 0°$ amp and $V_2 = 25\angle 0°$ volt.

(ii) With port-1 open, a voltage of $100\angle 0°$ volt is applied to port-2, resulted in, $I_2 = 20\angle 0°$ amp and $V_1 = 50\angle 0°$ volt.

(a) Write the loop equations for the network and also find the driving point and transfer impedance.

(b) What will be the voltage across a 10 Ω resistor connected across port-2 if a $100\angle 0°$ volt source is connected across port-1.

Solution

(a) From the given data, we get the *z*-parameters as:

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{100\angle 0°}{10\angle 0°} = 10\ \Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{25\angle 0°}{10\angle 0°} = 2.5\ \Omega$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{50\angle 0°}{20\angle 0°} = 2.5\ \Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{100\angle 0^\circ}{20\angle 0^\circ} = 5\ \Omega$$

So, the loop equations are:

$$\left.\begin{aligned} V_1 &= 10I_1 + 2.5I_2 \\ V_2 &= 2.5I_1 + 5I_2 \end{aligned}\right\}$$

(b) Here, $V_1 = 100\angle 0^\circ$ and $V_2 = -I_2 R_L = -10I_2$

Putting these values in loop equations,

$$100 = 10I_1 + 2.5I_2 \quad \Rightarrow \quad I_1 = 10 - 0.25I_2$$

and $\quad -10I_2 = 2.5I_1 + 5I_2$

or, $\quad -10I_2 = 2.5(10 - 0.25I_2) + 5I_2$

or, $\quad -15I_2 = 25 - 0.625I_2$

or, $\quad I_2 = \dfrac{-25}{14.375} = -1.74$ A

$\therefore$ Voltage across the resistor $= -I_2R_L = 17.4$ V

7.6 (a) The following equations give the voltages V_1 and V_2 at the two ports of a two port network, $V_1 = 5I_1 + 2I_2$, $\quad V_2 = 2I_1 + I_2$;
A load resistance of 3 Ω is connected across port-2. Calculate the input impedance.

(b) The z-parameters of a two port network are $z_{11} = 5\ \Omega$, $z_{22} = 2\ \Omega$, $z_{12} = z_{21} = 3\ \Omega$. Load resistance of 4 Ω is connected across the output port. Calculate the input impedance.

Solution

(a) From the given equations,

$$V_1 = 5I_1 + 2I_2 \qquad \text{(i)}$$

$$V_2 = 2I_1 + I_2 \qquad \text{(ii)}$$

At the output, $V_2 = -I_2R_L = -3I_2$

Putting this value in (ii),

$$-3I_2 = 2I_1 + I_2 \quad \Rightarrow \quad I_2 = -I_1/2$$

Putting in (i), $V_1 = 5I_1 + \left(\dfrac{-I_1}{2}\right) = 4I_1$

$\therefore$ Input impedance, $Z_{\text{in}} = \dfrac{V_1}{I_1} = 4\Omega$

(b) [Same as Prob. (a)] $Z_{\text{in}} = \dfrac{V_1}{I_1} = 3.5\Omega$

7.7 Determine the h-parameter with the following data:

(i) with the output terminals short circuited, $V_1 = 25$ V, $I_1 = 1$ A, $I_2 = 2$ A

(ii) with the input terminals open circuited, $V_1 = 10$ V, $V_2 = 50$ V, $I_2 = 2$ A

Solution The h-parameter equations are,

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

(a) <u>With output short-circuited</u>, $V_2 = 0$, given: $V_1 = 25$ V, $I_1 = 1$ A and $I_2 = 2$ A.

$$\therefore \quad \left.\begin{aligned} 25 &= h_{11} \times 1 \\ \text{and} \quad 2 &= h_{21} \times 1 \end{aligned}\right\} \Rightarrow h_{11} = 25\ \Omega, \text{ and } h_{21} = 2$$

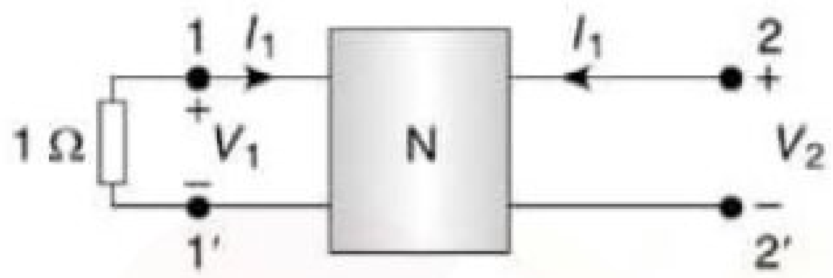
(b) <u>With input open-circuited</u>, $I_1 = 0$, given: $V_1 = 10$ V, $V_2 = 50$ V and $I_2 = 2$ A.

$$\therefore \quad \left.\begin{aligned} 10 &= h_{12} \times 50 \\ \text{and} \quad 2 &= h_{22} \times 50 \end{aligned}\right\} \Rightarrow h_{12} = \frac{1}{5} = 0.2 \text{ and } h_{23} = \frac{1}{25}\mho = 0.04\ \mho$$

Thus, the h-parameters are:

$$[h] = \begin{bmatrix} 25\ \Omega & 0.2 \\ 2 & 0.04\ \Omega^{-1} \end{bmatrix}$$

7.8 The y-parameters for a two-port network N are given as, $[y_{11} = 4\ \mho, y_{22} = 5\ \mho, y_{12} = y_{21} = 4\ \mho]$
If a resistor of 1 ohm is connected across port-1 of N, then find out the output impedance.



Solution Output impedance is given as,

$$Z_{out} = \frac{z_{11}z_{22} - z_{12}z_{21}}{z_{11} + Z_L}$$

Here, $y_{11} = 4\ \Omega^{-1}$, $y_{12} = y_{21} = 4\ \Omega^{-1}$, $y_{22} = 5\ \Omega^{-1}$

$$\therefore \quad z_{11} = \frac{y_{22}}{\Delta y} = \frac{5}{20 - 16} = \frac{5}{4}\ \Omega$$

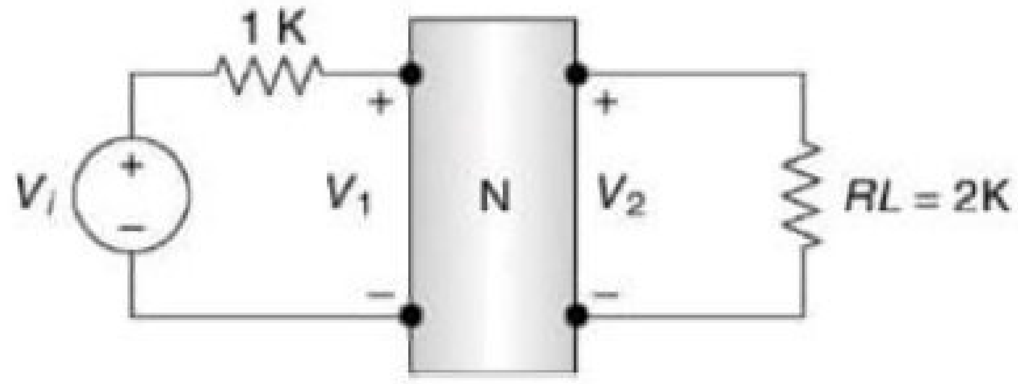
$$z_{12} = z_{21} = -\frac{y_{12}}{\Delta y} = -\frac{4}{4} = -1\ \Omega$$

$$\text{and} \quad z_{22} = \frac{y_{11}}{\Delta y} = \frac{4}{4} = 1\ \Omega$$

Putting these values,

$$Z_{out} = \frac{z_{11}z_{22} - z_{12}z_{21}}{z_{11} + Z_L} = \frac{\frac{5}{4} \times 1 - (-1) \times (-1) + 1 \times 1}{5/4 + 1} = \frac{5}{9}\ \Omega$$

7.9 (a) The h-parameters of a two-port network are $h_{11} = 100\ \Omega$, $h_{12} = 0.0025$, $h_{21} = 20$ and $h_{22} = 1$ m℧. Find V_2/V_1.



(b) The h-parameters of a two-port network are $h_{11} = 1\ \Omega$, $h_{12} = -h_{21} = 2$, $h_{22} = 1\ \mho$. The power absorbed by a load resistance of 1 Ω connected across port-2 is 100 W. The network is excited by a voltage source of generated voltage V_s and internal resistance 2 Ω. Calculate the value of V_s.

Solution

(a) The h-parameter equations are:

$$V_1 = 100I_1 + 0.0025V_2 \qquad \text{(i)}$$

$$I_2 = 20I_1 + 0.001V_2 \quad \text{(ii)}$$

By KVL at the output mesh, $V_2 = -2000I_2$ (iii)

$$V_1 = 100\left[\frac{I_2 - 0.001V_2}{20}\right] + 0.0025V_2 = 5\left(-\frac{V_2}{2000}\right) - 0.005V_2 + 0.0025V_2$$

From (i),

or $\quad \frac{V_2}{V_1} = -200$

(b) The h-parameter equations are:

$$V_1 = I_1 + 2V_2 \quad \text{(i)}$$

$$I_2 = -2I_1 + V_2 \quad \text{(ii)}$$

Since the load resistance of 1 Ω is connected across port-2,

$\therefore \quad \frac{V_2^2}{1} = 100 \Rightarrow V_2 = 10$ V

By KVL, $\quad V_2 = -I_2R_L = -I_2 \Rightarrow I_2 = -10$ A

and $\quad 2I_1 + V_1 = V_s$ (iii)

From (ii), putting the values of I_2 and V_2,

$$-10 = -2I_1 + 10 \Rightarrow I_1 = 10 \text{ A}$$

From (iii),

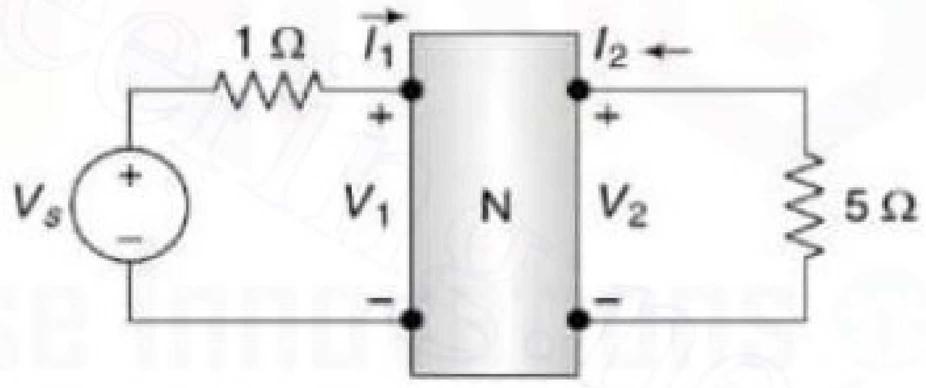
$$V_s = 2 \times 10 + V_1 = 20 + I_1 + 2V_2 \quad \{\text{by (i)}\}$$

$$= 20 + 10 + 2 \times 10$$

or, $\quad V_s = 50$ V

7.10 The z-parameters for a network N are:

$$\begin{bmatrix} 2 & 1 \\ 2 & 5 \end{bmatrix}$$



The terminal connections for the network are shown in the adjacent figure. Calculate the voltage ratio V_2/V_s, current ratio $-I_2/I_1$ and input resistance V_1/I_1.

Solution The z-parameter equations are:

$$V_1 = 2I_1 + I_2 \quad \text{(i)}$$

$$V_2 = 2I_1 + 5I_2 \quad \text{(ii)}$$

By KVL at the input and output circuits,

$$I_1 + V_1 = V_s \Rightarrow 3I_1 + I_2 = V_s \quad \text{(iii)} \quad \{\text{by (i)}\}$$

and $\quad 5I_2 + V_2 = 0 \Rightarrow 2I_1 + 10I_2 = 0 \quad$ (iv) {by(ii)}

Solving (iii) and (iv),

$$I_1 = \frac{\begin{vmatrix} V_s & 1 \\ 0 & 10 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 2 & 10 \end{vmatrix}} = \frac{10}{28}V_s \quad \text{and} \quad I_2 = \frac{\begin{vmatrix} 3 & V_s \\ 2 & 0 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 2 & 10 \end{vmatrix}} = -\frac{2}{28}V_s$$

$\therefore \quad -\frac{I_2}{I_1} = \frac{1}{5}$

Now, $\quad V_2 = (2I_1 + 5I_2) = \left(\frac{20}{28} - \frac{10}{28}\right)V_s = \frac{10}{28}V_s$

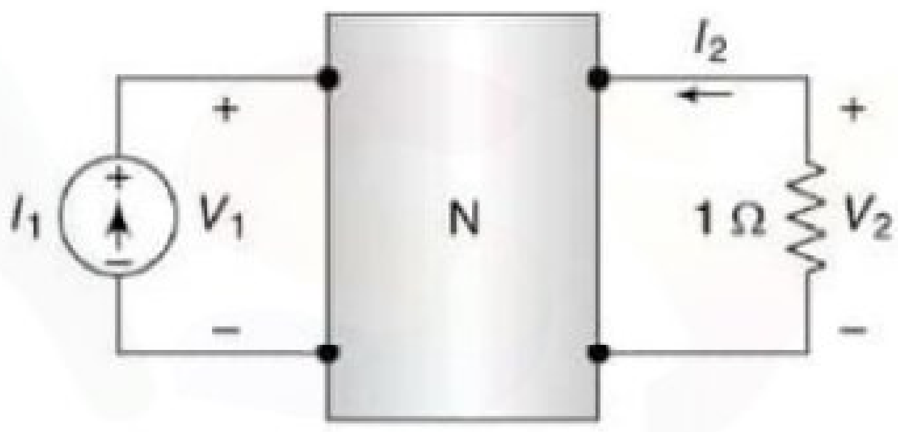
$\therefore \quad \frac{V_2}{V_s} = \frac{5}{14}$

Again,

$$V_1 = (2I_1 + I_2) = \left(\frac{20}{28} - \frac{2}{28}\right)V_s = \frac{18}{28}V_s$$

$\therefore \quad \frac{V_1}{I_1} = \frac{9}{14}\,\Omega$

7.11 For the two-port network in figure, terminated in a 1 Ω resistance, show that, $\frac{V_2}{I_1} = \frac{z_{21}}{1+z_{22}}$ and $\frac{V_1}{I_1} = \frac{z_{11}+\Delta z}{1+z_{22}}$



Solution The z-parameter equations are:

$$V_1 = z_{11}I_1 + z_{12}I_2 \qquad \text{(i)}$$
$$V_2 = z_{21}I_1 + z_{22}I_2 \qquad \text{(ii)}$$

By KVL at the output, $V_2 = -I_2 \times 1 \Rightarrow I_2 = -V_2$

$$V_2 = z_{21}I_1 + z_{22}I_2 = z_{21}I_1 + z_{22}(-V_2)$$

From (ii), or, $V_2(1+z_{22}) = z_{21}I_1$ (iii)

or $\quad \frac{V_2}{I_1} = \frac{z_{21}}{1+z_{22}}$ (Proved)

From (i),

$$V_1 = z_{11}\left[\frac{V_2(1+z_{22})}{z_{21}}\right] + z_{12}(-V_2) \quad \{\text{by (iii)}\}$$

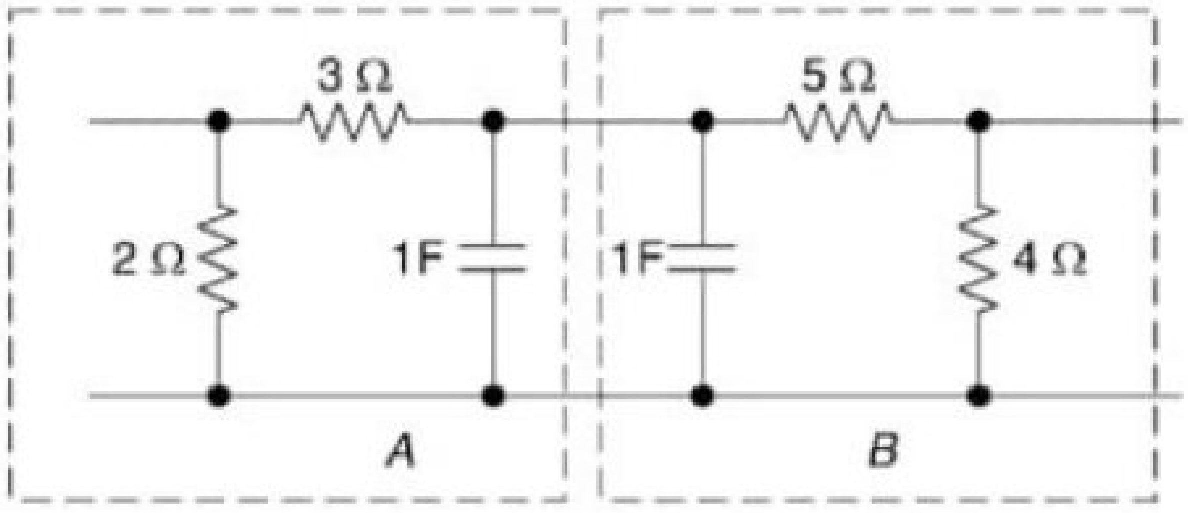
$$= V_2\left[\frac{z_{11} + z_{11}z_{22} - z_{12}z_{21}}{z_{21}}\right]$$

$$= V_2\left[\frac{z_{11}+\Delta z}{z_{21}}\right]$$

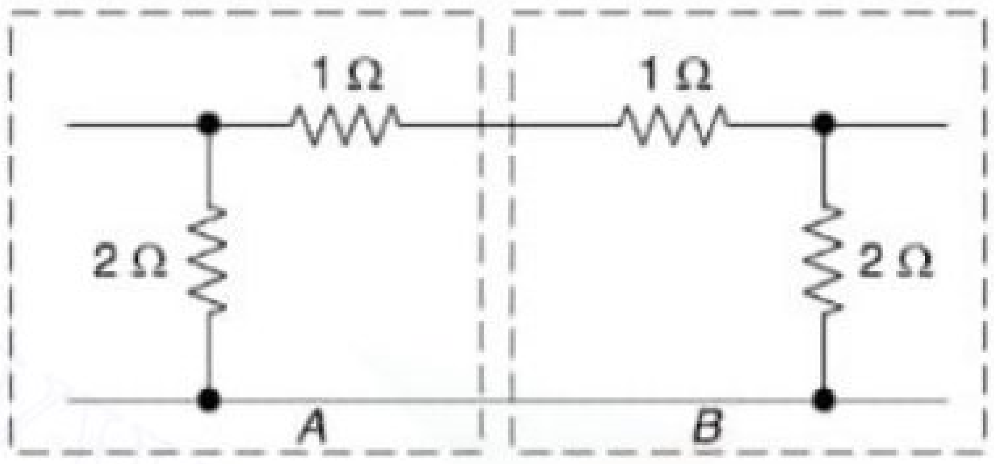
$\therefore \quad \frac{V_1}{I_1} = \frac{V_1}{V_2} \times \frac{V_2}{I_1} = \frac{z_{11}+\Delta z}{z_{21}} \times \frac{z_{21}}{1+z_{22}} = \frac{z_{11}+\Delta z}{1+z_{22}}$ (Proved)

7.12 Calculate the T-parameters for the block A and B separately and then using these results, calculate the T-parameters of the whole circuit shown in the figure. Prove any formula used.

(a)



(b)



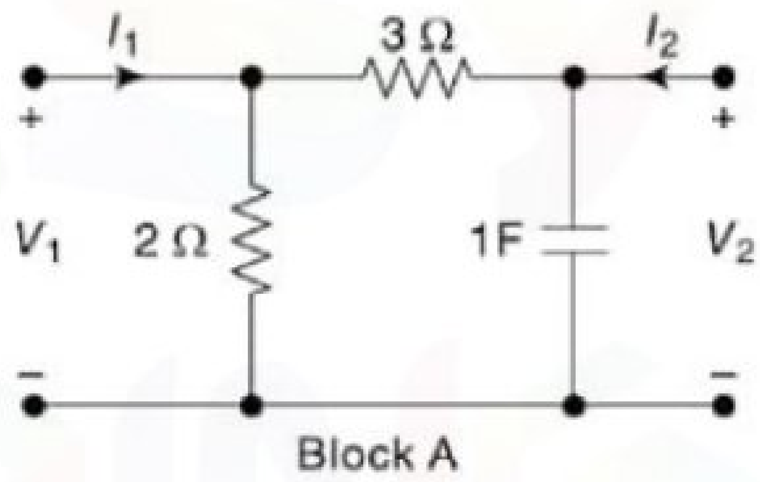
Solution

(a) We consider the given network as a cascade connection of two networks as shown.

For Block A:

Opening the port-2,

By KCL,



$$\left(\frac{1}{2}+\frac{1}{3}\right)V_1-\frac{1}{3}V_2=I_1$$

and $$-\frac{1}{3}V_1+\left(\frac{1}{3}+s\right)V_2=0$$

Solving for V_1 and V_2,

$$V_1=\frac{2I_1(1+3s)}{(1+5s)} \quad \text{and} \quad V_2=\frac{2I_1}{(1+5s)}$$

$$\therefore \quad A_a=\left.\frac{V_1}{V_2}\right|_{I_2=0}=(1+3s)$$

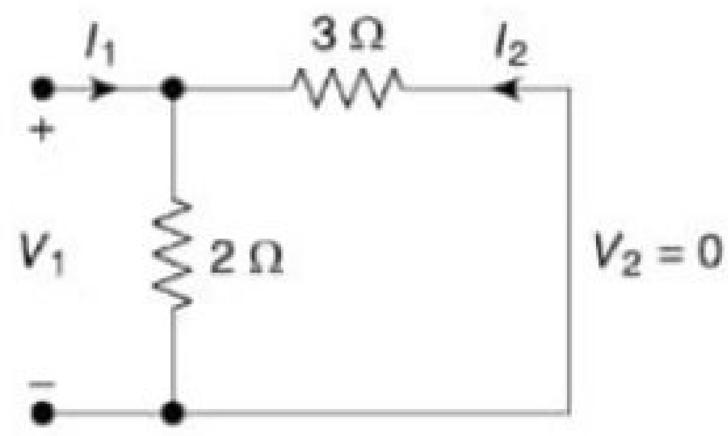
and $$C_a=\left.\frac{I_1}{V_2}\right|_{I_2=0}=\frac{(1+5s)}{2}$$

Short-circuiting port-2,

$$\therefore \quad I_1=\frac{V_1}{2}+\frac{V_1}{3}=\frac{5}{6}V_1$$

and $$V_1=-3I_2 \Rightarrow B_a=-\left.\frac{V_1}{I_2}\right|_{V_2=0}=3\Omega$$

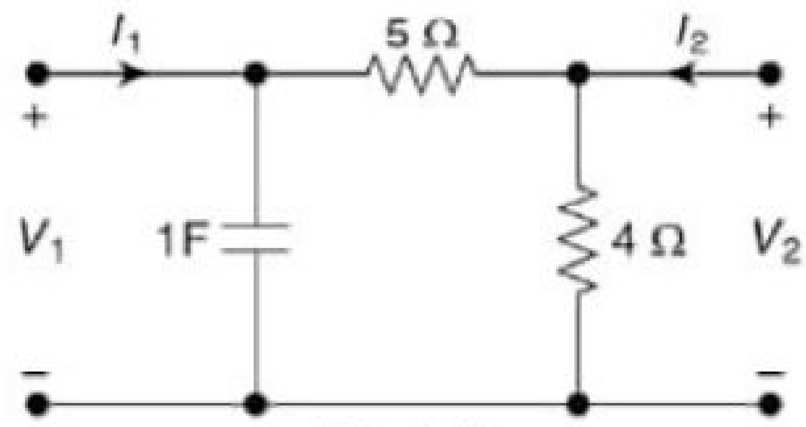
and $$D_a=-\left.\frac{I_1}{I_2}\right|_{V_2=0}=\frac{5V_1}{6}\times\frac{3}{V_1}=\frac{5}{2}$$



For Block B:

Opening the port-2,

By KCL,



Block B

$$\left(\frac{1}{5}+s\right)V_1-\frac{1}{5}V_2=I_1$$

and $$-\frac{1}{5}V_1+\left(\frac{1}{5}+\frac{1}{4}\right)V_2=0$$

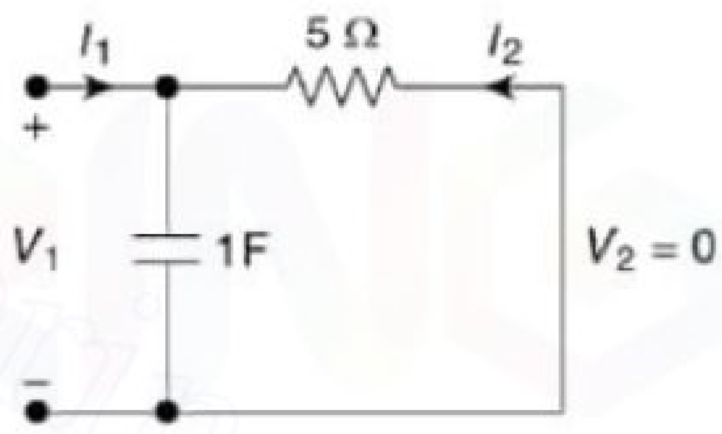
Solving for V_1 and V_2,

$$V_1=\frac{9I_1}{(1+9s)} \quad \text{and} \quad V_2=\frac{4I_1}{(1+9s)}$$

$$\therefore \quad A_b=\left.\frac{V_1}{V_2}\right|_{I_2=0}=\frac{9}{4}$$

and $$C_b=\left.\frac{I_1}{V_2}\right|_{I_2=0}=\frac{(1+9s)}{4}$$

Short-circuiting port-2,



$$\therefore \quad I_1=\left(\frac{1}{5}+s\right)V_1$$

and $$V_1=-5I_2 \Rightarrow B_b=-\left.\frac{V_1}{I_2}\right|_{V_2=0}=5\,\Omega$$

and $$D_b=-\left.\frac{I_1}{I_2}\right|_{V_2=0}=(5s+1)$$

Since the two networks are connected in cascade, the overall transmission parameter matrix is obtained as,

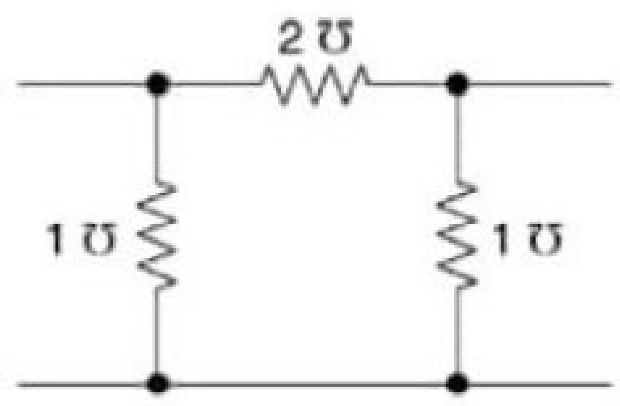
$$[T]=[T_a]\times[T_b]=\begin{bmatrix}(3s+1) & 3\\ \left(\frac{5s+1}{2}\right) & 5/2\end{bmatrix}\times\begin{bmatrix}9/4 & 5\\ \left(\frac{1+9s}{4}\right) & (5s+1)\end{bmatrix}=\begin{bmatrix}(13.5s+3) & (30s+8)\\ (11.25s+1.75) & (25s+5)\end{bmatrix}$$

(b) [Same as Prob. (a)]

Here, $$[T_a]=\begin{bmatrix}1 & 1\\ 1/2 & 3/2\end{bmatrix} \quad \text{and} \quad [T_b]=\begin{bmatrix}3/2 & 1\\ 3/2 & 1\end{bmatrix}$$

$$\therefore \quad [T]=[T_a]\times[T_b]=\begin{bmatrix}3 & 2\\ 3 & 2\end{bmatrix}$$

7.13 Two identical sections of the network shown in the figure are connected in parallel. Obtain the y-parameters of the resulting network and verify by direct calculation.



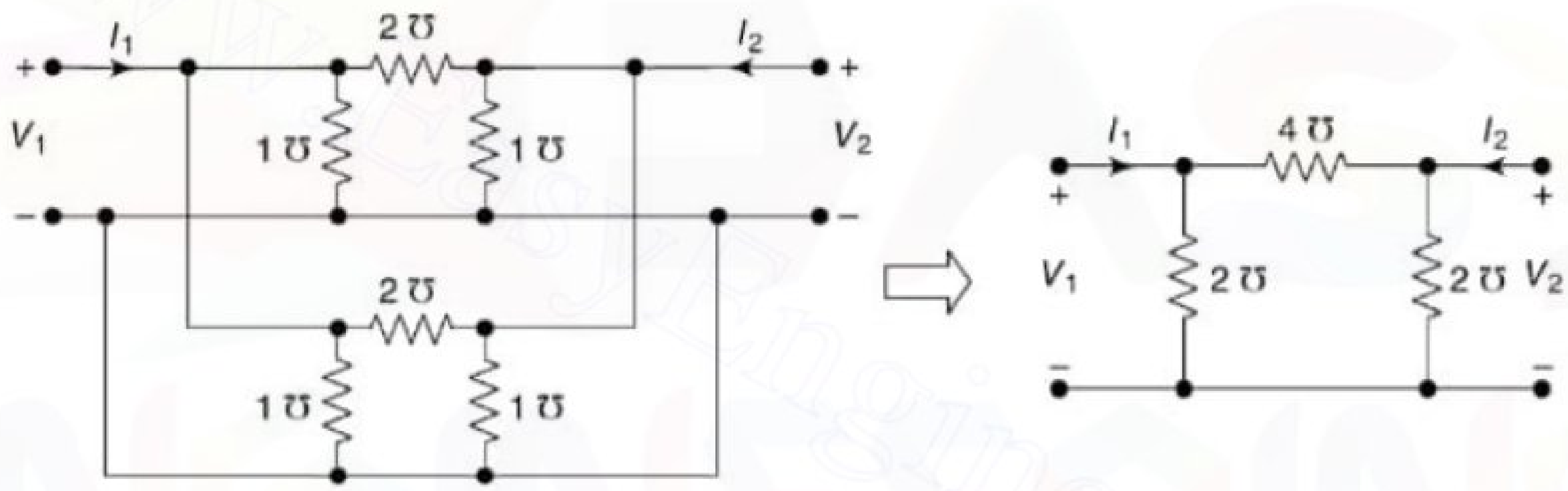
Solution For the circuit, $y_{11} = 3\,\Omega^{-1}$, $y_{12} = y_{21} = -2\,\Omega^{-1}$ and $y_{22} = 3\Omega^{-1}$

The y-parameters for the combination will be,

$$\left.\begin{array}{l} y_{11} = (y'_{11} + y''_{11}) = 6\,\Omega^{-1} \\ y_{12} = y_{21} = (y'_{12} + y''_{12}) = -4\,\Omega^{-1} \\ y_{22} = (y'_{22} + y''_{22}) = 6\,\Omega^{-1} \end{array}\right\}$$

To find the y-parameters by direct calculation, we consider the resulting network as shown.

For the entire network, $y_{11} = 4 + 2 = 6\,\Omega^{-1}$; $y_{12} = y_{21} = -4\,\Omega^{-1}$; $y_{22} = 4 + 2 = 6\,\Omega^{-1}$ (Proved)



7.14 Two networks have general *ABCD* parameters as shown below:

Parameter	*Network-1*	*Network-2*
A	1.50	5/3
B	11Ω	4Ω
C	0.25 siemens	1 siemens
D	2.5	3.0

If the two networks are connected with their inputs and outputs in parallel, obtain the admittance matrix of the resulting network.

Solution *For network-1:*

$$y_{11} = \frac{D}{B} = \frac{2.5}{11} = \frac{5}{22}\,\Omega^{-1}$$

$$y_{12} = -\frac{AD - BC}{B} = -\frac{1.5 \times 2.5 - 11 \times 0.25}{11} = -\frac{1}{11}\,\Omega^{-1}$$

$$y_{21} = -\frac{1}{B} = -\frac{1}{11}\,\Omega^{-1}$$

$$y_{22} = \frac{A}{B} = \frac{1.5}{11} = \frac{3}{22}\,\Omega^{-1}$$

For network-2:

$$y_{11} = \frac{D}{B} = \frac{3}{4}\,\Omega^{-1}$$

$$y_{12} = -\frac{AD - BC}{B} = -\frac{1}{4}\,\Omega^{-1}$$

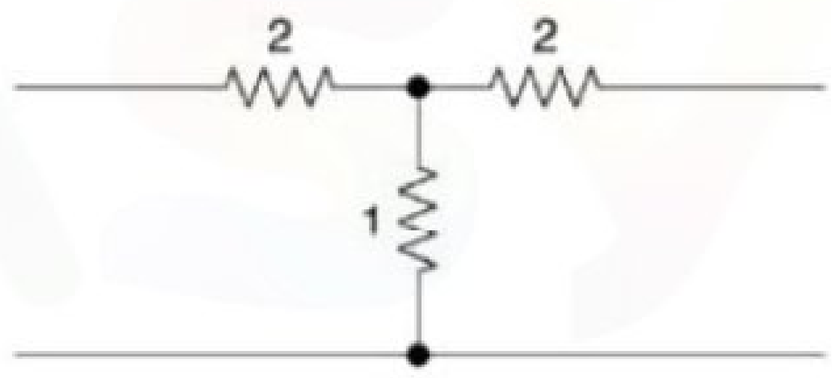
$$y_{21} = -\frac{1}{B} = -\frac{1}{4}\,\Omega^{-1}$$

$$y_{22} = \frac{A}{B} = \frac{5}{3\times 4} = \frac{5}{12}\,\Omega^{-1}$$

So, the admittance matrix of the resulting network is:

$$[y] = \begin{bmatrix} 5/22 & -1/11 \\ -1/11 & 3/22 \end{bmatrix} + \begin{bmatrix} 3/4 & -1/4 \\ -1/4 & 5/12 \end{bmatrix} = \begin{bmatrix} 43/44 & -15/44 \\ -15/44 & 73/132 \end{bmatrix}\Omega^{-1}$$

7.15 Two identical sections of figure are connected in series. Obtain the z-parameters of the resulting network and verify by direct calculation. All values are in ohm.



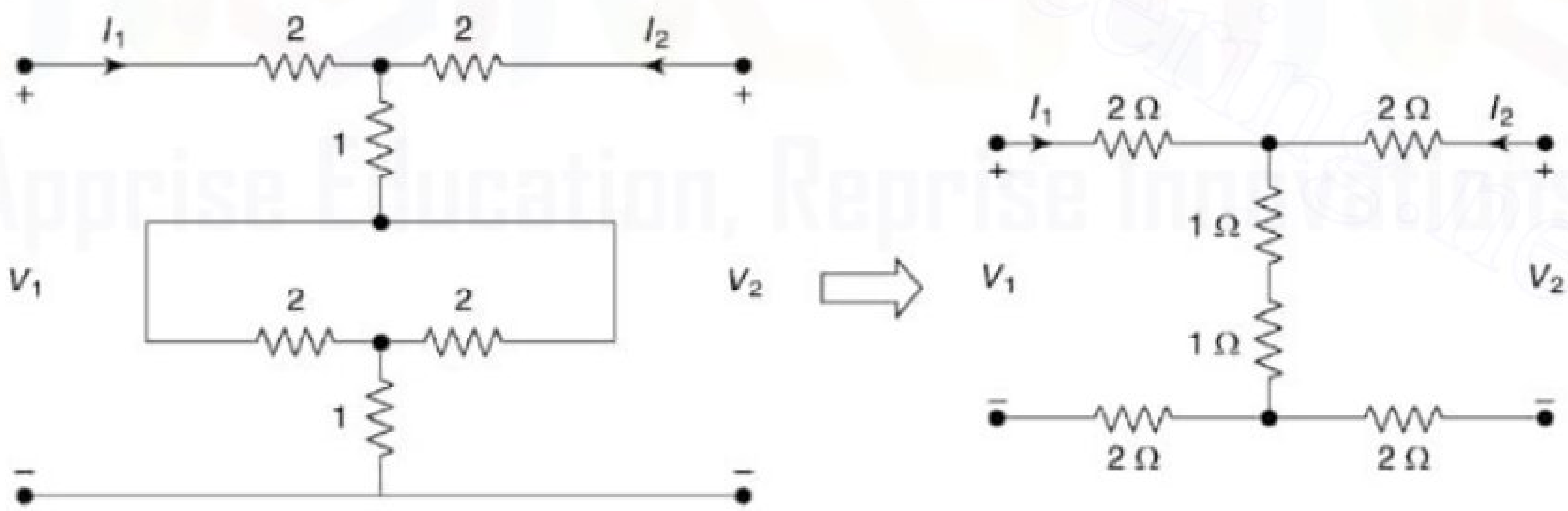
Solution The z-parameters of each section:

$$z_{11} = 3\,\Omega,\ z_{12} = z_{21} = 1\,\Omega,\ z_{22} = 3\,\Omega$$

So, the z-parameters of the combined series network are:

$$z_{11} = (3+3) = 6\,\Omega,\ z_{12} = z_{21} = (1+1) = 2\,\Omega,\ z_{22} = (3+3) = 6\,\Omega$$

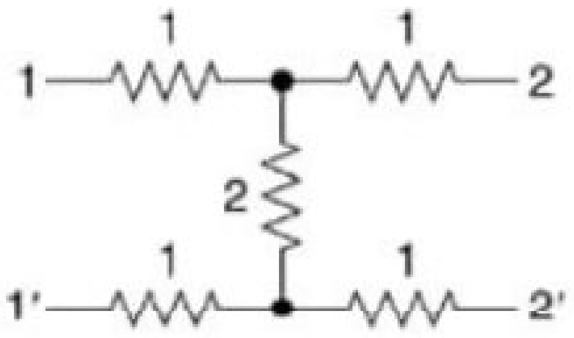
To find the z-parameters by direct calculation, we consider the resulting network as shown.



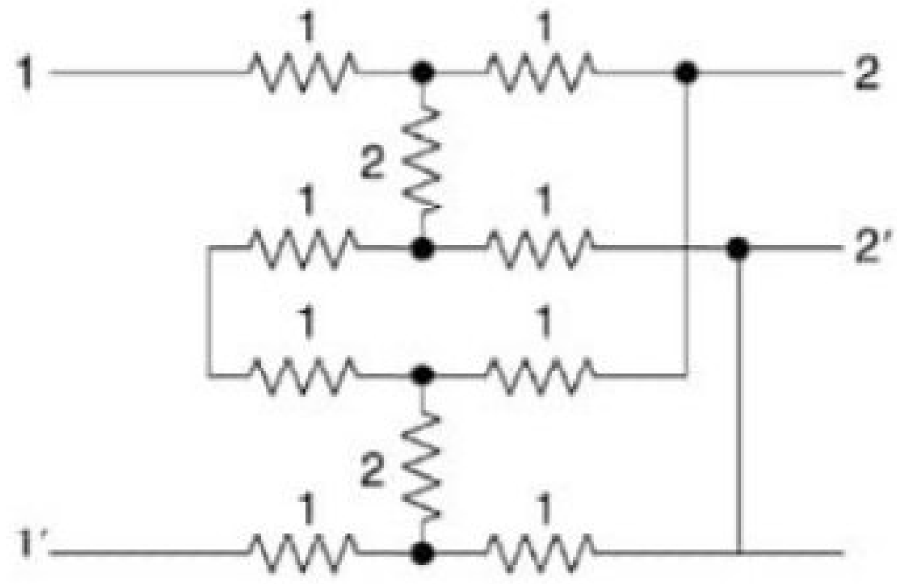
For the resulting network,

$$\left.\begin{aligned} z_{11} &= \left.\frac{V_1}{I_1}\right|_{I_2=0} = 6\,\Omega & z_{21} &= \left.\frac{V_2}{I_1}\right|_{I_2=0} = 2\,\Omega \\ z_{22} &= \left.\frac{V_2}{I_2}\right|_{I_1=0} = 6\,\Omega & z_{12} &= \left.\frac{V_1}{I_2}\right|_{I_1=0} = 2\,\Omega \end{aligned}\right\}$$

7.16 (a) Find out the z- and h-parameters for the circuit shown in Fig. (a). All values are in ohm.
(b) Hence, obtain the hybrid parameters for the two-port network of Fig. (b).



(a)



(b)

Solution

(a) For Fig. (a), the z-parameters are:

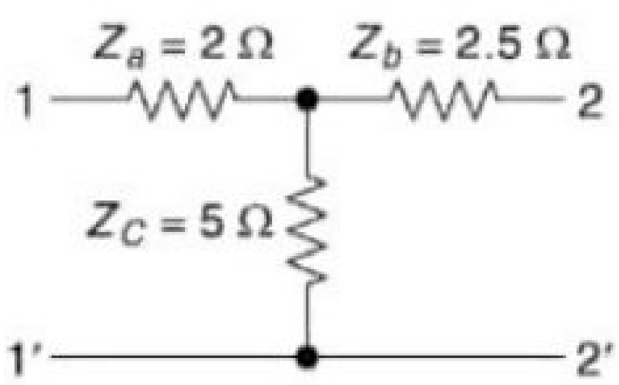
$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 4\ \Omega,\ z_{12} = z_{21} = 2\ \Omega,\ z_{11} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 4\ \Omega$$

$$\therefore \quad \left.\begin{aligned} h_{11} &= \frac{\Delta z}{z_{12}} = \frac{16-4}{4} = 3\ \Omega \\ h_{12} &= \frac{z_{12}}{z_{22}} = \frac{2}{4} = 0.5 \\ h_{21} &= -\frac{z_{21}}{z_{22}} = -\frac{2}{4} = -0.5 \\ h_{22} &= \frac{1}{z_{12}} = \frac{1}{4} = 0.25\ \Omega^{-1} \end{aligned}\right\}$$

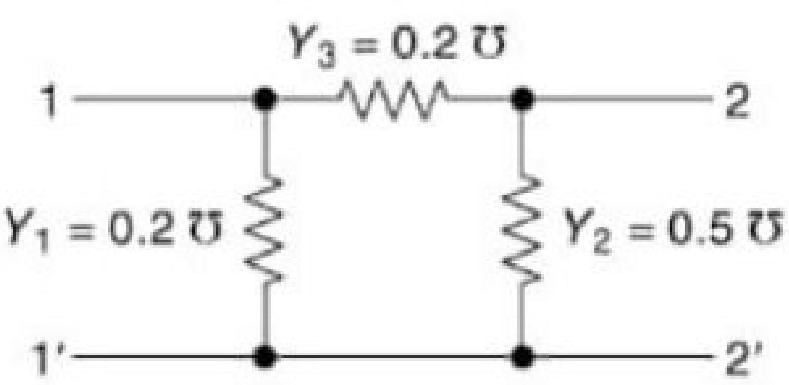
(b) The connection is series-parallel connection. For this connection, the overall h-parameters will be the sum of individual h-parameters.

$$\therefore \quad \left.\begin{aligned} h_{11} &= (3+3) = 6\Omega \\ h_{12} &= (0.5+0.5) = 1 \\ h_{21} &= (-0.5-0.5) = -1 \\ h_{22} &= (0.25+0.25) = 0.5\Omega^{-1} \end{aligned}\right\}$$

7.17 (a) Find the equivalent π-network for the T-network shown in the Fig. (a).
(b) Find the equivalent T-network for the π-network shown in the Fig. (b).



(a)



(b)

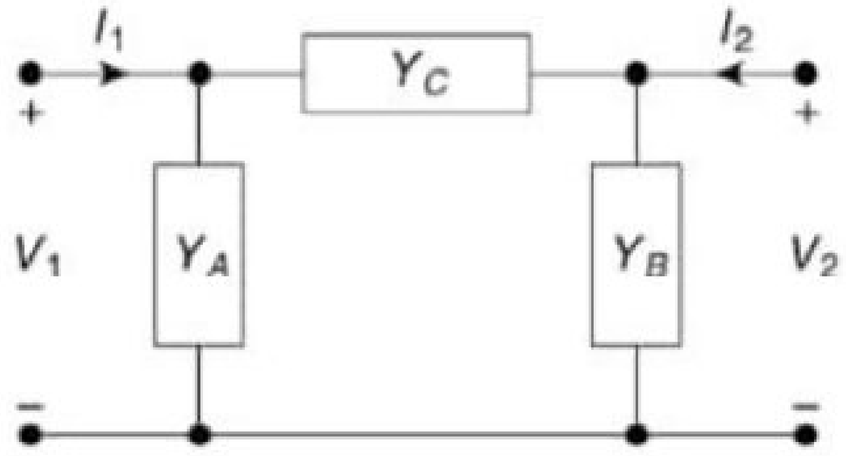
Solution

(a) Let the equivalent π-network have Y_C as the series admittance and Y_A and Y_B as the shunt admittances at port-1 and port-2, respectively.

Now, the z-parameters are given as:

$$z_{11} = (Z_A + Z_C) = 7\ \Omega,\ z_{12} = z_{21} = Z_C = 5\ \Omega,\ z_{22} = (Z_B + Z_C) = 7.5\ \Omega$$

$$\therefore \quad \Delta z = (7 \times 7.5 - 5 \times 5) = 27.5\ \Omega^2$$



$$\therefore \quad y_{11} = \frac{z_{22}}{\Delta z} = \frac{7.5}{27.5}\ \mho$$

$$y_{12} = y_{21} = -\frac{z_C}{\Delta z} = -\frac{5}{27.5}\ \mho$$

$$y_{22} = \frac{z_{11}}{\Delta z} = \frac{7}{27.5}\ \mho$$

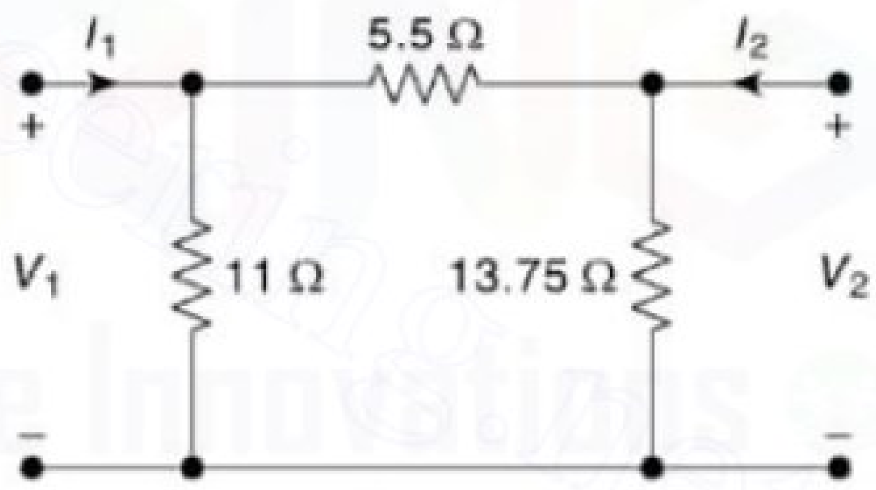
$$\therefore \quad Y_A = (y_{11} + y_{12}) = \frac{2.5}{27.5} = \frac{1}{11}\ \mho$$

$$\therefore \quad Y_B = (y_{22} + y_{12}) = \frac{2}{27.5}\ \mho$$

$$\text{and} \quad Y_C = -y_{21} = \frac{5}{27.5} = \frac{2}{11}\ \mho$$

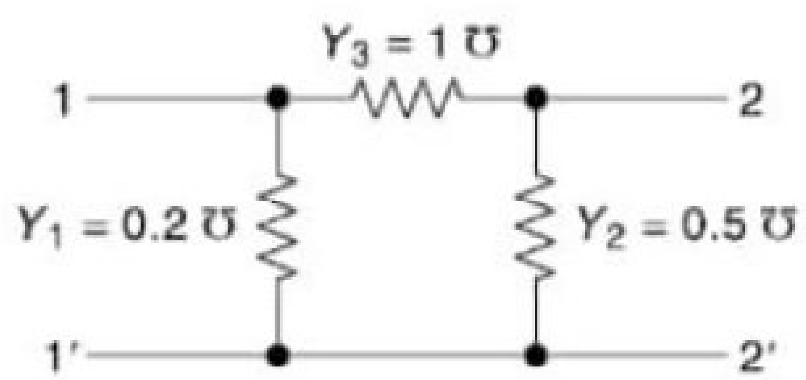
Thus, the impedances of the equivalent π-networks are:

$$\left.\begin{aligned} Z_A &= \frac{1}{Y_A} = 11\ \Omega, \\ Z_B &= \frac{1}{Y_B} = 13.75\ \Omega, \\ Z_C &= \frac{1}{Y_C} = 5.5\ \Omega \end{aligned}\right\}$$

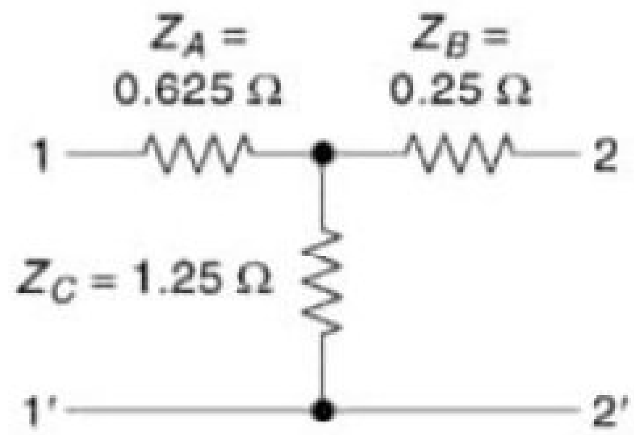


Equivalent π-network

(b)



π-network



Equivalent T-network

The y-parameters,

$$y_{11} = 1.2\ \mho,\ y_{12} = y_{21} = -1\ \mho, \text{ and } y_{22} = 1.5\ \mho$$

$$\therefore \quad \Delta y = (1.2 \times 1.5 - 1) = 0.8$$

$$\therefore \quad z_{11} = \frac{y_{22}}{\Delta y} = \frac{1.5}{0.8}\,\Omega,\ z_{12} = z_{21} = -\frac{y_{12}}{\Delta y} = \frac{1}{0.8}\,\Omega,\ z_{22} = \frac{y_{11}}{\Delta y} = \frac{1.2}{0.8}\,\Omega$$

$$\therefore \quad \left.\begin{aligned} Z_A &= (z_{11} - z_{12}) = \frac{0.5}{0.8} = 0.625\,\Omega \\ Z_B &= (z_{22} - z_{12}) = \frac{0.2}{0.8} = 0.25\,\Omega \\ Z_C &= z_{12} = \frac{1}{0.8} = 1.25\,\Omega \end{aligned}\right\}$$

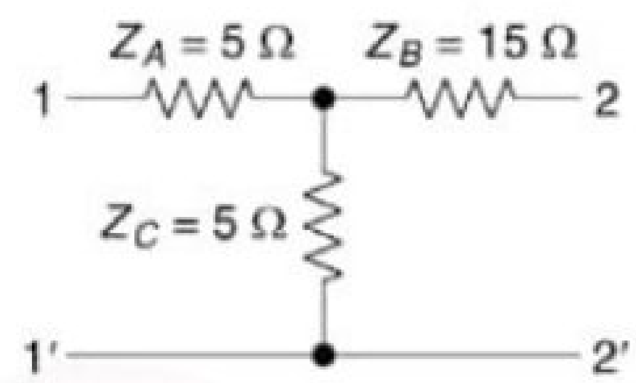
7.18 The z-parameter of a 2-port network are:

$$z_{11} = 10\,\Omega,\ z_{22} = 20\,\Omega,\ z_{12} = z_{21} = 5\,\Omega.$$

Find the $ABCD$-parameters. Also find the equivalent T-network.

Solution

From the inter-relationship, we get the $ABCD$ parameters as:



Equivalent T-network

$$A = \frac{z_{11}}{z_{21}} = \frac{10}{5} = 2$$

$$B = \frac{z_{11}Z_{22} - Z_{12}Z_{21}}{z_{21}} = \frac{10 \times 20 - 5 \times 5}{5} = 35\,\Omega$$

$$C = \frac{1}{z_{21}} = \frac{1}{5} = 0.2\,\mho$$

$$D = \frac{z_{22}}{z_{21}} = \frac{20}{5} = 4$$

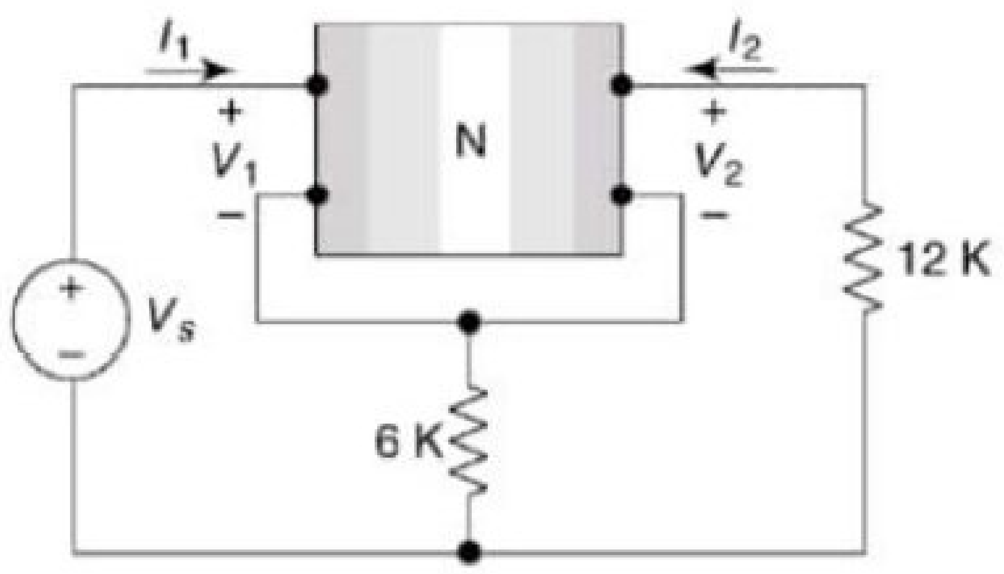
To find the equivalent T-network, we have the relations,

$$\left.\begin{aligned} z_{11} &= (Z_A + Z_C) = 10\,\Omega \\ z_{12} &= z_{21} = Z_C = 5\,\Omega \\ \text{and} \quad z_{22} &= (Z_B + Z_C) = 20\,\Omega \end{aligned}\right\} \Rightarrow Z_A = 5\,\Omega,\ Z_B = 15\,\Omega,\ Z_C = 5\,\Omega$$

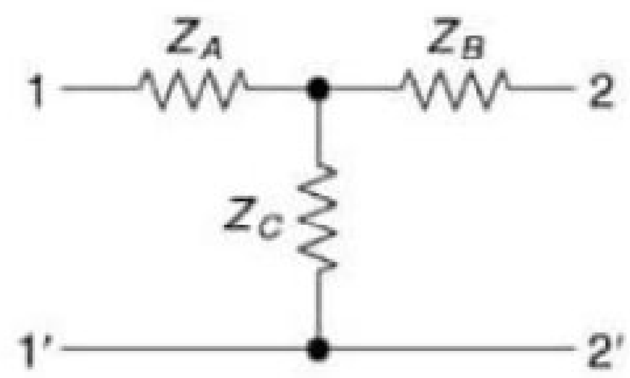
7.19 Z-parameters of the two-port network N in figure. are, $z_{11} = 4s$, $z_{12} = z_{21} = 3s$, $z_{22} = 9s$.

(a) Replace N by its T-equivalent.

(b) Use part (a) to find the input current I_1 for $V_s = \cos 1000t$.



Solution



Equivalent T-network

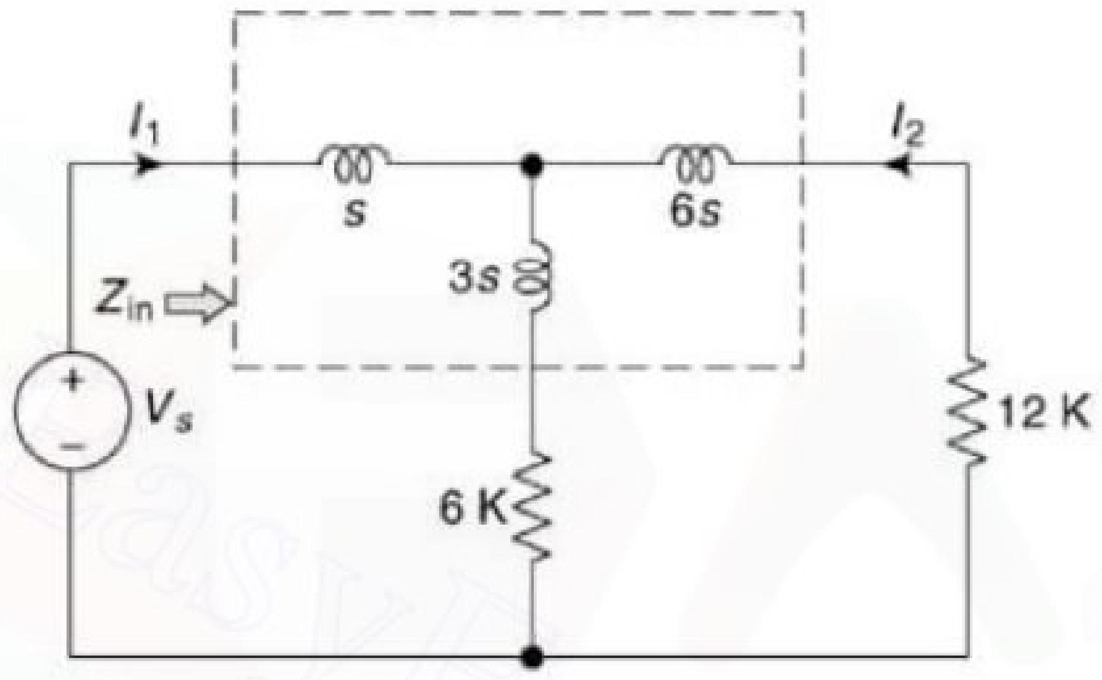
(a) The z-parameters are: $[z]=\begin{bmatrix}4s & 3s\\ 3s & 9s\end{bmatrix}(\Omega)$

Since the network is reciprocal, its *T*-equivalent exists. Its elements are:

$$Z_A = (z_{11} - z_{12}) = s,\ Z_B = (z_{22} - z_{21}) = 6s,$$

and $\quad Z_C = z_{21} = z_{12} = 3s$

So, the equivalent circuit is shown in figure.



(b) We repeatedly combine the series and parallel elements of above figure, with resistors in kΩ and *s* in Krad/s to find the input impedance, Z_{in} in kΩ.

$$\therefore \qquad Z_{in} = \frac{V_s}{I_1} = s + \frac{(6s+12)(3s+6)}{(6s+12)+(3s+6)} = (3s+4)$$

or $\quad Z_{in}(j) = (3j+4) = 5\angle 36.9^\circ$ kΩ

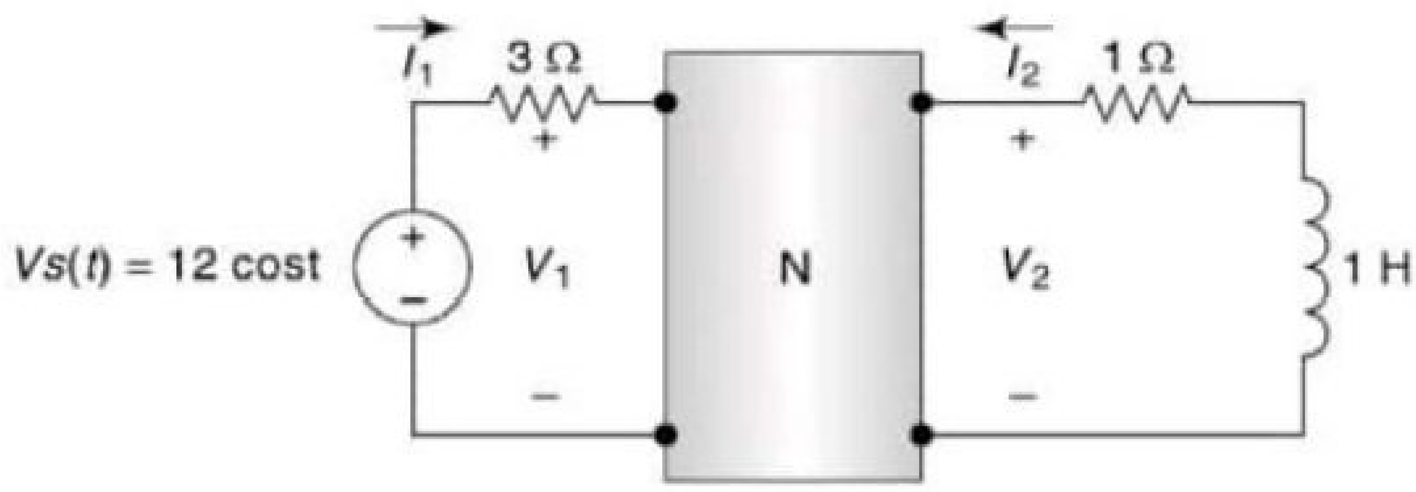
So, the current,

$$i(t) = \frac{v_s(t)}{Z_{in}(j)} = \frac{1}{5}\cos(1000t - 36.9^\circ)\ (\text{mA})$$

7.20 The *z*-parameters of a two-port network *N* are given by, $z_{11} = (2s + 1/s)$, $z_{12} = z_{21} = 2s$, $z_{22} = (2s + 4)$.

(a) Find the *T*-equivalent of *N*.

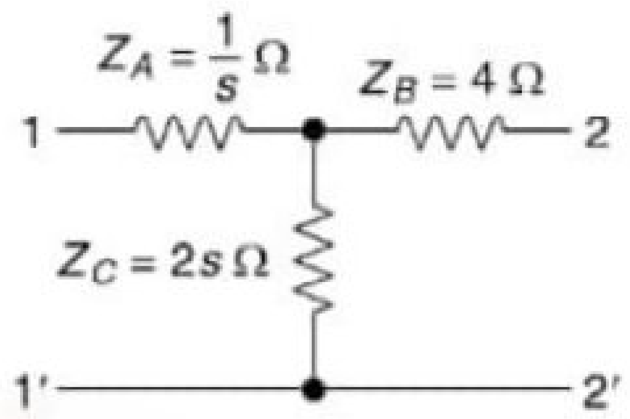
(b) The network *N* is connected to a source and a load as shown in figure. Replace *N* by its *T*-equivalent and then find I_1, I_2, V_1, and V_2.



Solution

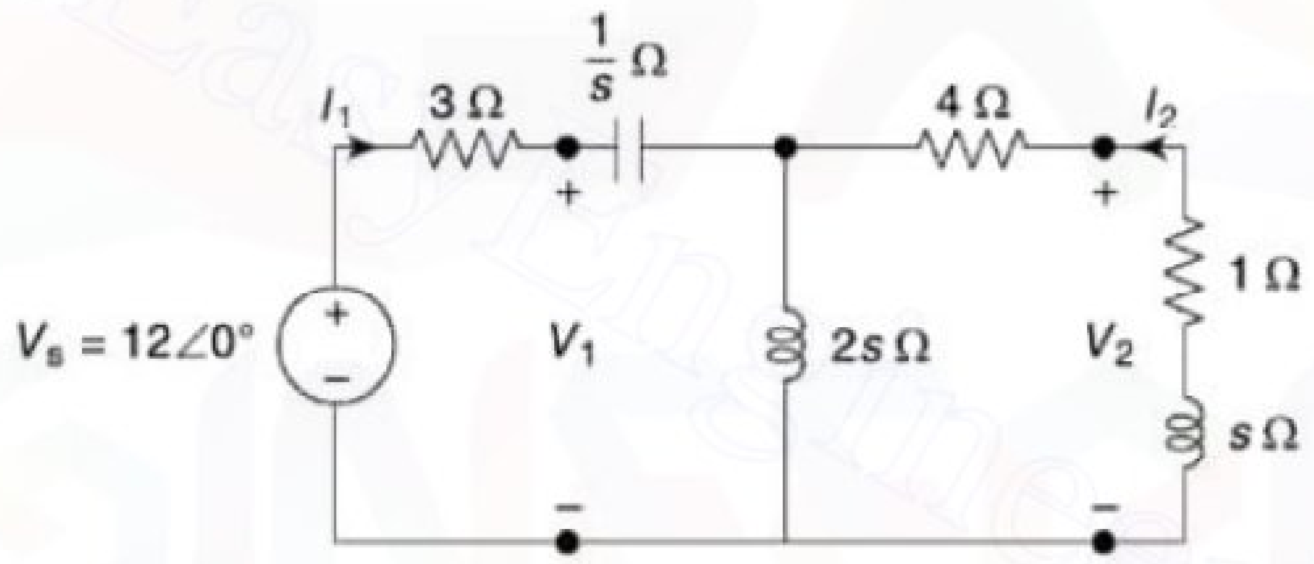
(a) To find the equivalent *T*-network, we have the relations,

$$\left.\begin{aligned} z_{11} &= (Z_A + Z_C) = \left(2s + \frac{1}{s}\right)\Omega \\ z_{12} &= z_{21} = Z_C = 2s\ \Omega \\ \text{and} \quad z_{22} &= (Z_B + Z_C) = (2s + 4)\ \Omega \end{aligned}\right\} \Rightarrow Z_A = \frac{1}{s}\Omega,\ Z_B = 4\Omega,\ Z_C = 2s\ \Omega$$



Equivalent T-network

(b) The equivalent circuit is shown below.



By KVL, $I_1(3 + j) + I_2(j2) = 12\angle 0°$

$I_1(j2) + I_2(5 + j3) = 0$

$$I_1 = \frac{\begin{vmatrix} 12\angle 0° & j2 \\ 0 & (5+j3) \end{vmatrix}}{\begin{vmatrix} (3+j) & j2 \\ j2 & (5+j3) \end{vmatrix}} = \frac{\begin{vmatrix} 12\angle 0° & 2\angle 90° \\ 0 & 5.831\angle 30.96° \end{vmatrix}}{16 + j14} = 3.29\angle -10.22°\,(\text{A})$$

Solving,

$$\text{and} \qquad I_2 = \frac{\begin{vmatrix} (3+j) & 12\angle 0° \\ j2 & 0 \end{vmatrix}}{\begin{vmatrix} (3+j) & j2 \\ j2 & (5+j3) \end{vmatrix}} = 1.13\angle -131.19°\,(\text{A})$$

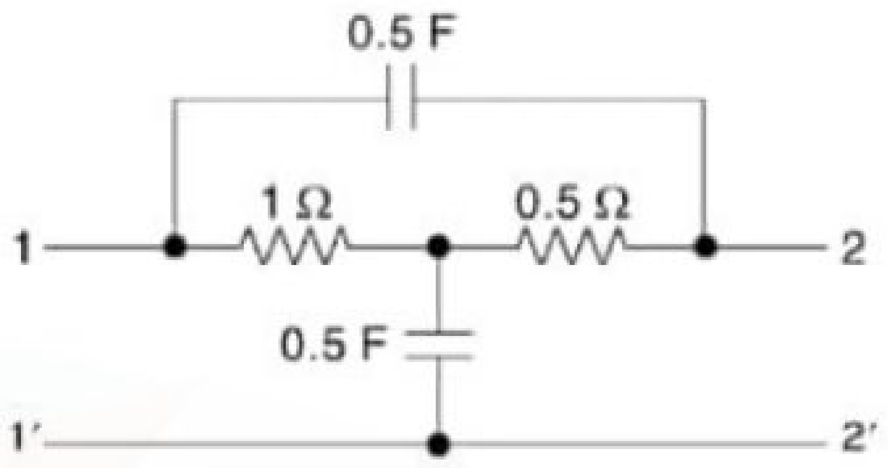
$\therefore \quad V_1 = 12\angle 0° - I_1 \times 3 = 12 - 3.29 \times 3\angle -10.22° = 2.28 + j1.75 = 2.88\angle 37.504°\ (\text{V})$

and $\quad V_2 = -I_2(1 + j) = -1.13(1 + j)\angle -131.186° = 1.59\angle 93.81°$

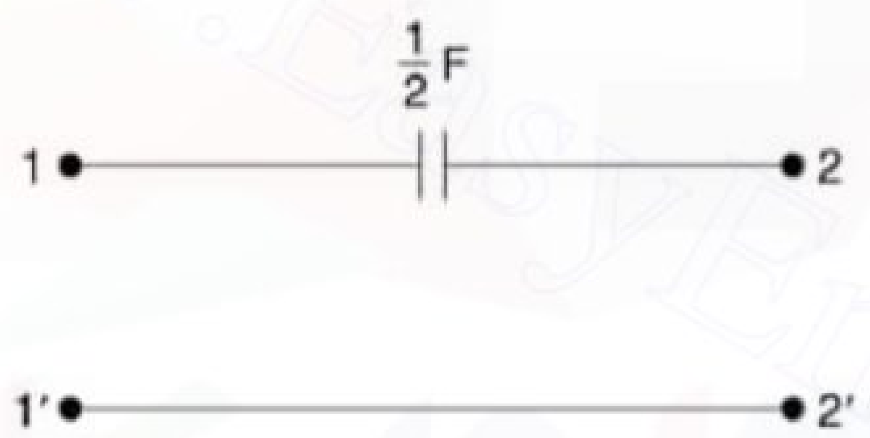
So, the currents and voltages are:

$$\left.\begin{aligned} i_1(t) &= 3.29\cos(t - 10.2°)\ (\text{A}) \\ i_2(t) &= 1.13\cos(t - 131.2°)\ (\text{A}) \\ v_1(t) &= 2.88\cos(t + 37.5°)\ (\text{A}) \\ v_2(t) &= 1.6\cos(t + 93.8°)\ (\text{A}) \end{aligned}\right\}$$

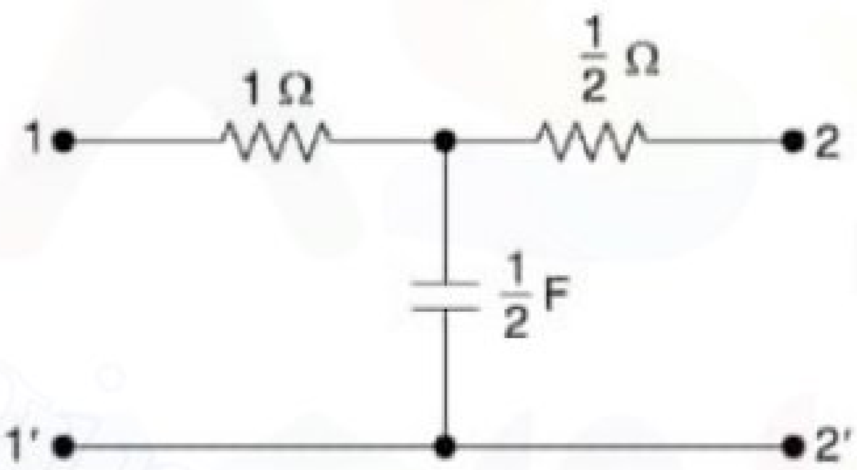
7.21 For the bridge-TRC network, find the y-parameters and its equivalent π-network.



Solution The given network is the parallel combination of the two networks:



(a) Network



(b) Network

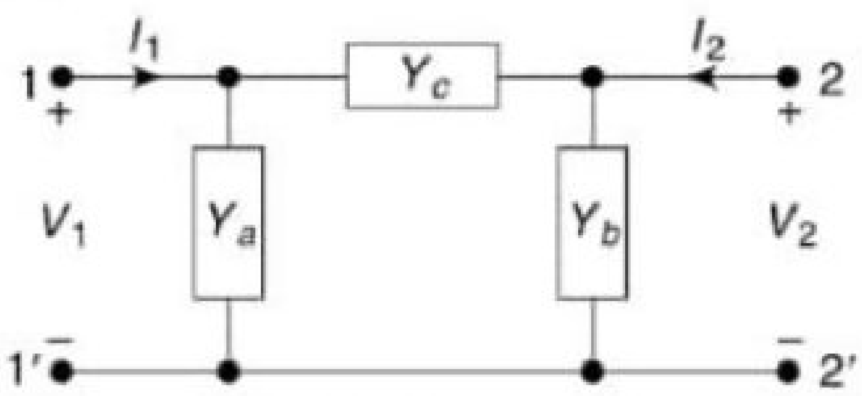
For network (a), the y-parameters are: $[y_a] = \begin{bmatrix} s/2 & -s/2 \\ -s/2 & s/2 \end{bmatrix}$ ℧

For network (b), the z-parameters are: $[z_b] = \begin{bmatrix} (1 + 2/s) & 2/s \\ 2/s & (1/2 + 2/s) \end{bmatrix} \Omega$

$$\therefore \qquad y_{11b} = \frac{z_{22b}}{\Delta z_b} = \frac{(1/2 + 2/s)}{(1 + 2/s)(1/2 + 2/s) - 4/s^2} = \frac{s+4}{s+6}$$

$$\therefore \qquad y_{12b} = y_{21b} = -\frac{z_{12b}}{\Delta z_b} = \frac{2/s}{(s+6)/2s} = \frac{4}{s+6}$$

$$\therefore \qquad y_{22b} = \frac{z_{11b}}{\Delta z_b} = \frac{(s+2)/2}{(s+6)/2s} = \frac{2(s+2)}{s+6}$$



Equivalent π network

For network (b), the y-parameters are: $[y_b] = \begin{bmatrix} \dfrac{s+4}{s+6} & \dfrac{4}{s+6} \\ \dfrac{4}{s+6} & \dfrac{2(s+2)}{s+6} \end{bmatrix}$

Thus, the overall y-parameters are:

$$[y] = [y_a] + [y_b] = \begin{bmatrix} s/2 & -s/2 \\ -s/2 & s/2 \end{bmatrix} + \begin{bmatrix} \dfrac{s+4}{s+6} & \dfrac{4}{s+6} \\ \dfrac{4}{s+6} & \dfrac{2(s+2)}{s+6} \end{bmatrix}$$

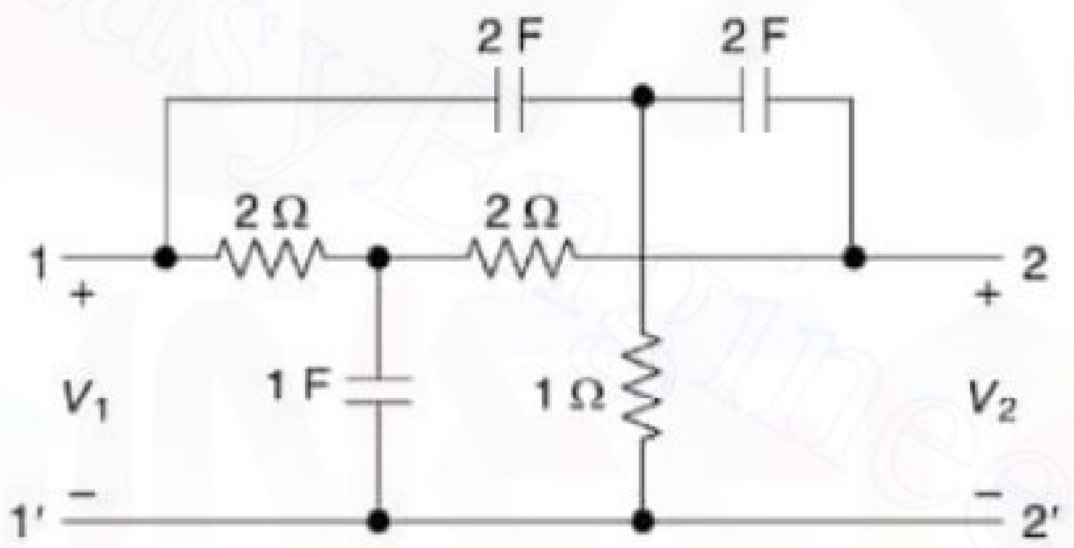
$$= \begin{bmatrix} \dfrac{s^2+8s+8}{2(s+6)} & -\dfrac{s^2+6s+8}{2(s+6)} \\ -\dfrac{s^2+6s+8}{2(s+6)} & \dfrac{s^2+10s+8}{2(s+6)} \end{bmatrix}$$

Equivalent π network can be found out from the relations:

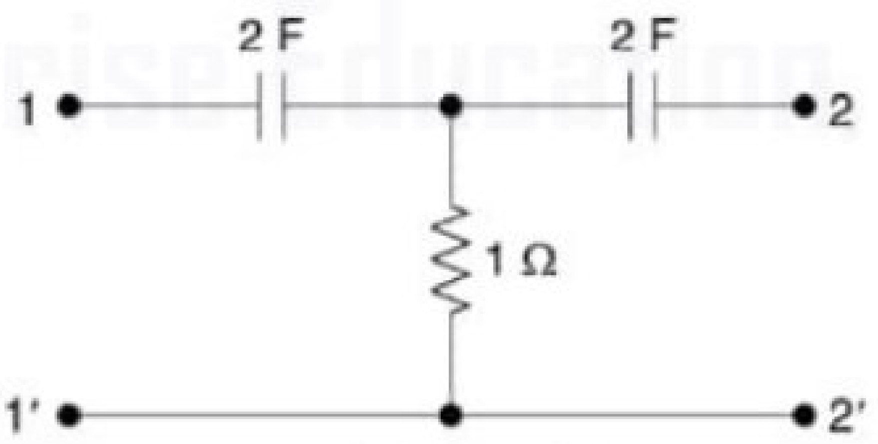
$$Y_a = (y_{11} + y_{12}) = \frac{s}{(s+6)}; \; Y_b = (y_{22} + y_{12})$$

$$= \frac{2s}{(s+6)}; \; Y_c = -y_{12} = -y_{21} = \frac{s^2+6s+8}{2(s+6)}$$

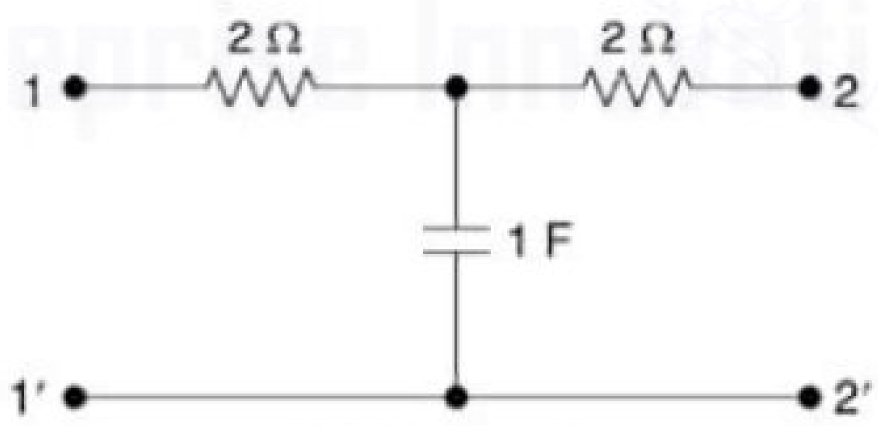
7.22 For the notch-filter network, determine the y-parameters.



Solution The given network is the parallel combination of the two networks:



(a) Network



(b) Network

For network (a), $z_{11a} = \left(\dfrac{1}{2s} + 1\right) = \dfrac{1+2s}{2s}; \; z_{12a} = z_{21a} = 1; \; z_{22a} = \left(\dfrac{1}{2s} + 1\right) = \dfrac{1+2s}{2s}$

$$\therefore \quad \Delta z_a = \frac{1+4s}{4s^2}$$

$$\therefore \quad y_{11a} = \frac{z_{22a}}{\Delta z_a} = \frac{2s(1+2s)}{(1+4s)}; \; y_{12a} = y_{21a} = -\frac{z_{12a}}{\Delta z_a}$$

$$= -\frac{4s^2}{(1+4s)}; \; y_{22a} = \frac{z_{11a}}{\Delta z_a} = \frac{2s(1+2s)}{(1+4s)}$$

For network (b), $z_{11b} = (1/s + 2) = \dfrac{1+2s}{s}$; $z_{12b} = z_{21b} = \dfrac{1}{s}$; $z_{22b} = (1/s + 2) = \dfrac{1+2s}{s}$

$$\therefore \quad \Delta z_b = \frac{4(s+1)}{s}$$

$$\therefore \quad y_{11b} = \frac{z_{22b}}{\Delta z_b} = \frac{(1+2s)}{4(s+1)};\; y_{12b} = y_{21b} = -\frac{z_{12b}}{\Delta z_b} = -\frac{1}{4(s+1)};\; y_{22b} = \frac{z_{11b}}{\Delta z_b} = \frac{(1+2s)}{4(s+1)}$$

Thus, the overall y-parameters are,

$$y_{11} = y_{22} = (y_{11a} + y_{11b}) = \frac{2s(1+2s)}{1+4s} + \frac{(1+2s)}{4+4s} = \frac{(1+2s)(8s^2+12s+1)}{4(s+1)(4s+1)}$$

and
$$y_{12} = y_{21} = (y_{12a} + y_{12b}) = -\frac{4s^2}{1+4s} - \frac{1}{4(s+1)} = -\frac{16s^3+16s^2+4s+1}{4(4s+1)(s+1)}$$

7.23 A network has two input terminals a, b and two output terminals c, d. The input impedance with c-d open-circuited is $(250 + j100)$ ohm and with c-d short-circuited is $(400 + j300)$ ohm. The impedance across c-d with a-b open-circuited is 200 ohm. Determine the equivalent T-network parameters.

Solution For c-d Terminals opened,

$$(Z_A + Z_B) = (250 + j100) \qquad \text{(i)}$$

But, for c-d terminals shorted,

$$Z_A + \frac{Z_B Z_C}{Z_B + Z_C} = (400 + j300) \qquad \text{(ii)}$$

Again, with a-b terminals opened,

$$(Z_B + Z_C) = 200 \qquad \text{(iii)}$$

From (ii) and (i), we get,

$$\frac{Z_B Z_C}{Z_B + Z_C} - Z_B = 150 + j200$$

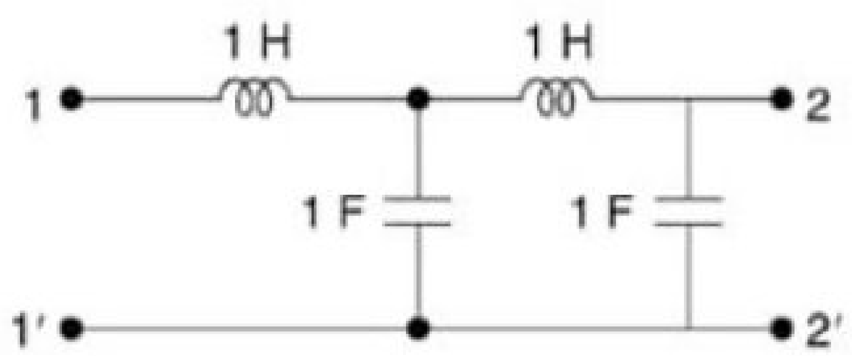
or
$$Z_B Z_C - Z_B^2 - Z_B Z_C = 200(150 + j200) \qquad \{\text{by (iii)}$$

or
$$Z_B^2 = 200(-150 - j200) = 10^4(1 - j2)^2$$

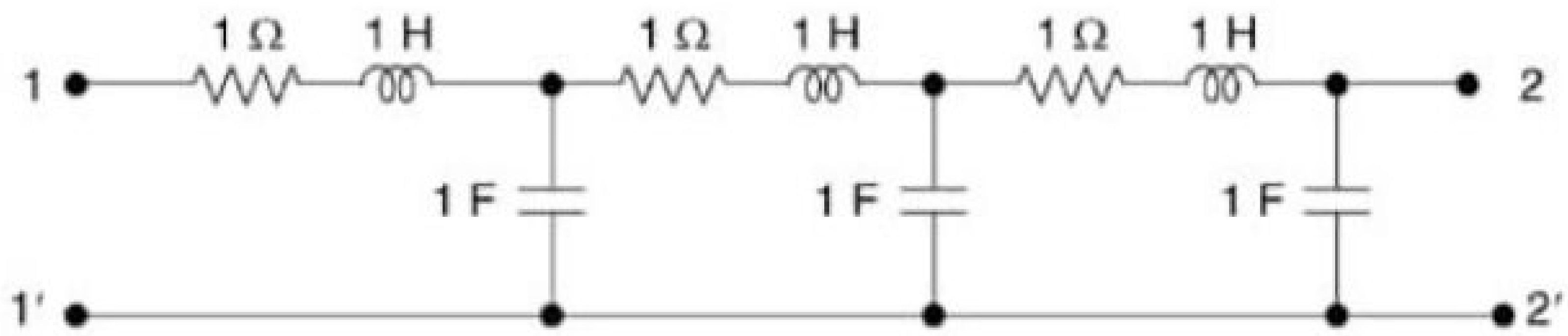
$$\left.\begin{aligned} \therefore \quad & Z_B = (100 - j200)\Omega \\ \therefore \quad & Z_A = (150 + j300)\Omega \\ \textit{and} \quad & Z_C = (100 + j200)\Omega \end{aligned}\right\}$$

7.24 Find the driving point impedance at the terminals 1-1′ of the ladder network shown in figure.

(a)



(b)



Solution

(a) The driving point impedance at 1-1′ is

$$Z_{11} = s + \cfrac{1}{s + \cfrac{1}{s + \cfrac{1}{s}}} = \frac{s^4 + 3s^2 + 1}{s^2 + 2s}$$

(b) The driving point impedance at 1-1′ is,

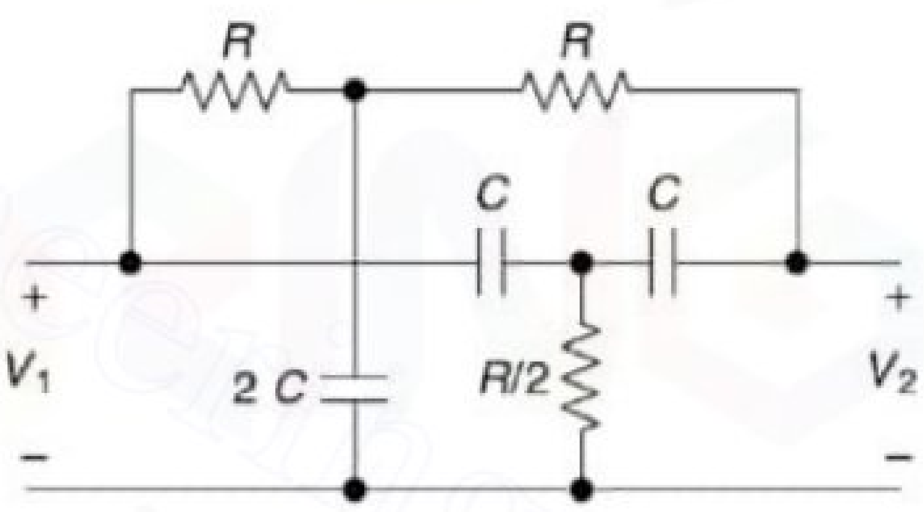
$$Z_{11} = (s+1) + \cfrac{1}{s + \cfrac{1}{(s+1) + \cfrac{1}{s + \cfrac{1}{(s+1) + \cfrac{1}{s}}}}} = \frac{s^6 + 3s^5 + 8s^4 + 11s^3 + 11s^2 + 6s + 1}{s^5 + 2s^4 + 5s^3 + 4s^2 + 3s}$$

7.25 For the Notch-filter (Twin-T) network, determine:

(a) y-parameters,

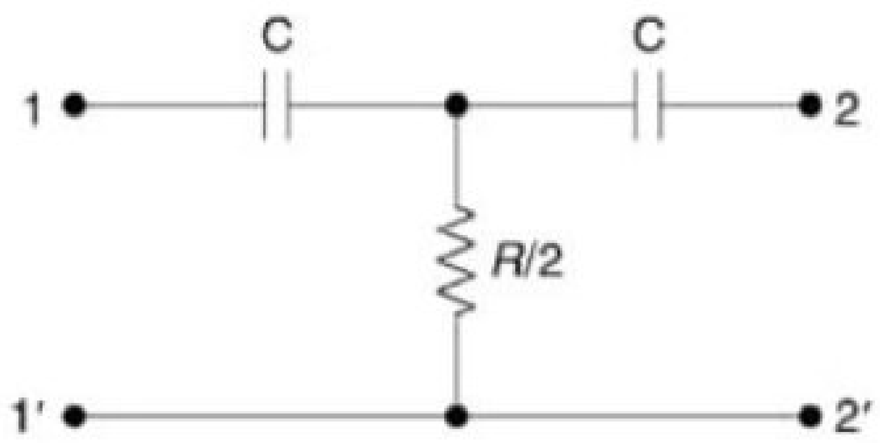
(b) the voltage ratio transfer function V_2/V_1 when no-load impedance is present, and

(c) the value of the frequency at which the output voltage is zero.

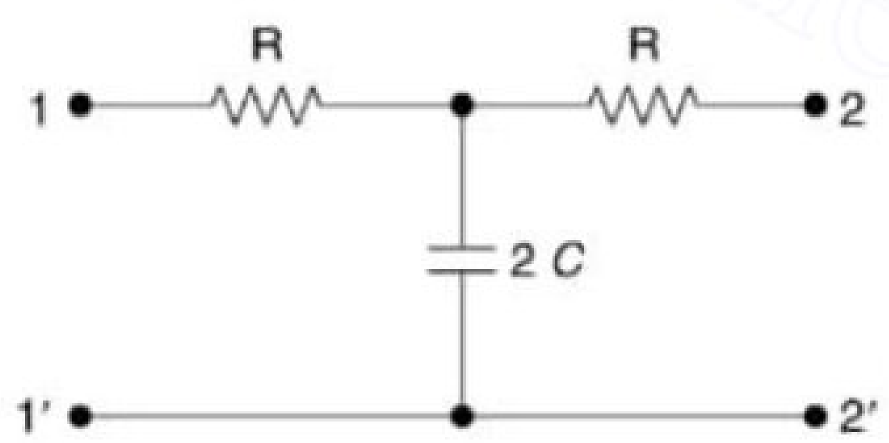


Solution

(a) The given network is the parallel combination of the two networks:



(a) Network



(b) Network

For network (a),

$$z_{11a} = \left(\frac{1}{Cs} + \frac{R}{2}\right) = \frac{2 + RCs}{2Cs};\ z_{12a} = z_{21a} = \frac{R}{2};\ z_{22a} = \left(\frac{1}{Cs} + \frac{R}{2}\right) = \frac{2 + RCs}{2Cs}$$

$$\therefore \qquad \Delta z_a = \frac{1 + RCs}{C^2 s^2}$$

$$\therefore \quad y_{11a} = \frac{z_{22a}}{\Delta z_a} = \frac{RCs(2+RCs)}{2R(1+RCs)}; \qquad y_{12a} = y_{21a} = -\frac{z_{12a}}{\Delta z_a} = -\frac{R^2C^2s^2}{2R(1+RCs)};$$

$$y_{22a} = \frac{z_{11a}}{\Delta z_a} = \frac{Cs\left(1+\frac{1}{2}Cs\right)}{(1+RCs)}$$

For network (b),

$$z_{11b} = \left(\frac{1}{2Cs}+R\right) = \frac{1+2RCs}{2Cs};\ z_{12b} = z_{21b} = \frac{1}{2Cs};\ z_{22b} = \left(\frac{1}{s}+2\right) = \frac{1+2RCs}{2Cs}$$

$$\therefore \quad \Delta z_b = \frac{1+RCs}{C^2s^2}$$

$$\therefore \quad y_{11b} = \frac{z_{22b}}{\Delta z_b} = \frac{(1+2RCs)}{2R(RCs+1)}; \qquad y_{12b} = y_{21b} = -\frac{z_{12b}}{\Delta z_b} = -\frac{1}{2R(RCs+1)};$$

$$y_{22b} = \frac{z_{11b}}{\Delta z_b} = \frac{(1+2RCs)}{2R(RCs+1)}$$

Thus, the overall y-parameters are,

$$y_{11} = y_{22} = (y_{11a}+y_{11b}) = \frac{RCs(2+RCs)}{2R(1+RCs)} + \frac{(1+2RCs)}{2R(RCs+1)} = \frac{(R^2C^2s^2+4RCs+1)}{2R(RCs+1)}$$

and

$$y_{12} = y_{21} = (y_{12a}+y_{12b}) = -\frac{R^2C^2s^2}{2R(1+RCs)} - \frac{1}{2R(RCs+1)} = -\frac{R^2C^2s^2+1}{2R(RCs+1)}$$

(b) Now,

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

When no-load impedance is present, $I_2 = 0$,

$$\therefore \quad \frac{V_2}{V_1} = -\frac{y_{21}}{y_{22}} = \frac{R^2C^2s^2+1}{2R(RCs+1)} \times \frac{2R(RCs+1)}{(R^2C^2s^2+4RCs+1)} = \frac{R^2C^2s^2+1}{(R^2C^2s^2+4RCs+1)}$$

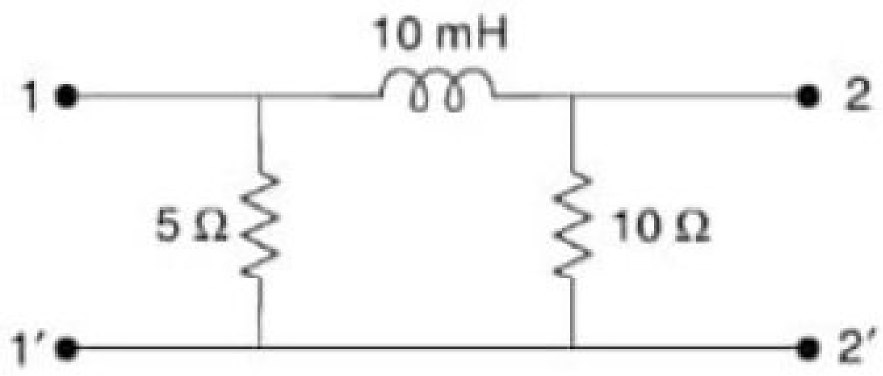
(c) For $V_2 = 0 \Rightarrow 1 + R^2C^2s^2 = 0$

Putting $s = j\omega$, $1 - \omega^2R^2C^2 = 0$

$$\therefore \quad \omega = \frac{1}{RC}$$

Thus, the notch frequency is given by, $f_N = \dfrac{1}{2\pi RC}$

7.26 Find the open circuit impedance parameters for the two-port network shown in the figure below.



Solution For this π-network, the y-parameters are given as,

$$y_{11} = \left(\frac{1}{5} + \frac{1}{0.01s}\right) = \left(0.2 + \frac{100}{s}\right);$$

$$y_{12} = y_{21} = -\frac{1}{0.01s} = -\frac{100}{s};$$

$$y_{22} = \left(\frac{1}{10} + \frac{1}{0.01s}\right) = \left(0.1 + \frac{100}{s}\right)$$

$$\therefore \quad \Delta y = (y_{11}y_{22} - y_{12}y_{21}) = \left(0.2 + \frac{100}{s}\right) \times \left(0.1 + \frac{100}{s}\right) - \left(-\frac{100}{s}\right)^2$$

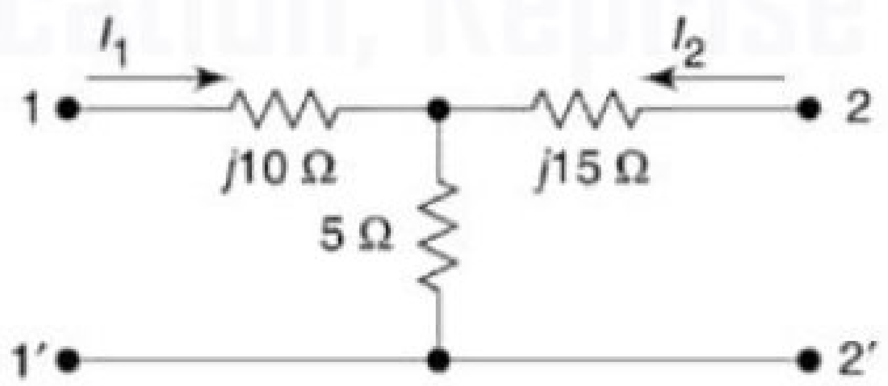
$$= 0.02 + \frac{30}{s} + \left(\frac{100}{s}\right)^2 - \left(-\frac{100}{s}\right)^2$$

$$= \left(0.02 + \frac{30}{s}\right)$$

Thus, the z-parameters are,

$$\left.\begin{aligned} z_{11} &= \frac{y_{22}}{\Delta y} = \frac{0.1 + 100/s}{0.02 + 30/s} = \frac{0.1s + 100}{0.02s + 30} = \frac{5s + 5000}{s + 1500}\ \Omega \\ z_{12} = z_{21} &= -\frac{y_{12}}{\Delta y} = -\frac{-100/s}{0.02 + 30/s} = \frac{100}{0.02s + 30} = \frac{5000}{s + 1500}\ \Omega \\ z_{22} &= \frac{y_{11}}{\Delta y} = \frac{0.2 + 100/s}{0.02 + 30/s} = \frac{0.2s + 100}{0.02s + 30} = \frac{10s + 5000}{s + 1500}\ \Omega \end{aligned}\right\} \quad \textit{Ans.}$$

7.27 Find the open-circuit impedance parameters of the circuit given in the figure. Also, find the h-parameters of the circuit.



Solution By KVL,

$$(j10 + 5)I_1 + 5I_2 = V_1 \qquad \text{(i)}$$

and

$$5I_1 + (j15 + 5)I_2 = V_2 \qquad \text{(ii)}$$

Thus, the z-parameters are:

$$z_{11} = (5 + j10)\ \Omega \quad z_{12} = z_{21} = 5\ \Omega \quad Z_{22} = (5 + j15)\ \Omega \qquad \textit{Ans.}$$

The hybrid parameter matrix may be written as

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

From Eq (ii), we get,

$$I_2 = -\frac{5}{5+j15}I_1 + \frac{V_2}{5+j15}$$

$$= -\frac{1}{1+j3}I_1 + \frac{1}{5+j15}V_2 \quad \text{(iii)}$$

Putting this value of I_2 in Eq (i), we get,

$$(5+j10)I_1 + 5\left[-\frac{5}{5+j15}I_1 + \frac{V_2}{5+j15}\right] = V_1$$

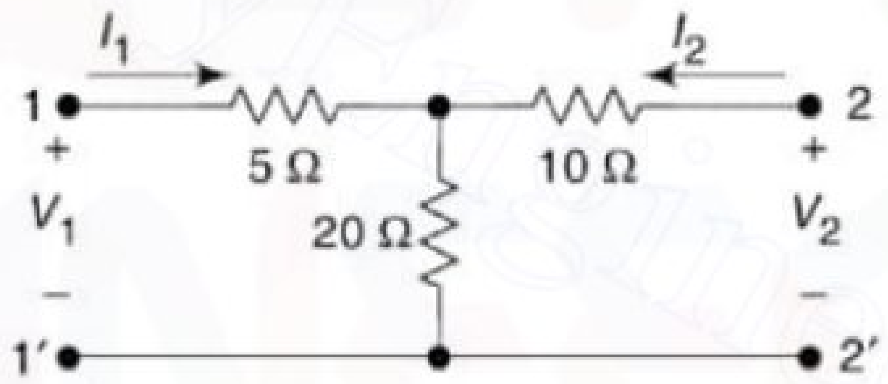
$$\Rightarrow \quad V_1 = \frac{(5+j10)\times(5+j15)-25}{(5+j15)}I_1 + \frac{5}{5+j15}V_2$$

$$= \frac{30+j25}{1+j3}I_1 + \frac{1}{1+j3} \quad \text{(iv)}$$

Comparing Eq (iii) and (iv) with the standard equations of h-parameters, we get,

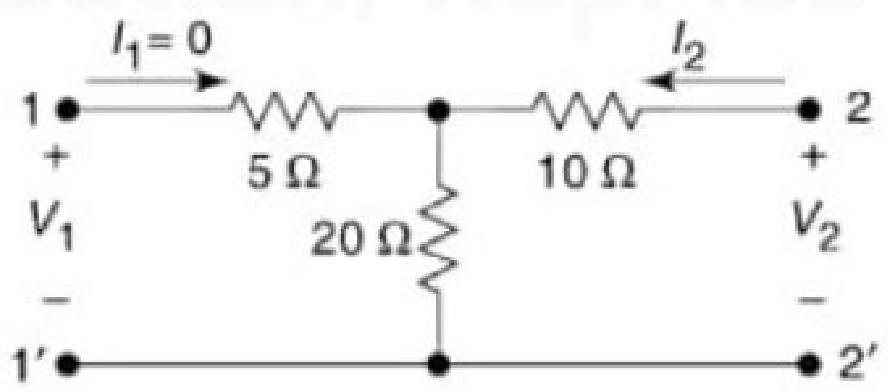
$$h_{11} = \frac{30+j25}{1+j3}\,\Omega;\quad h_{12} = \frac{1}{1+j3};\quad h_{21} = -\frac{1}{1+j3};\quad h_{22} = \frac{1}{5+j15}\,℧ \qquad \textit{Ans.}$$

7.28 Determine the z-parameters for the network shown in the figure.



Solution We consider two situations:

(a) When $I_1 = 0$, i. e. port-1 is open-circuited: In this case no current will flow through the 5Ω resistor.



Figure(a) *When $I_1 = 0$*

By KVL in the right mesh, we get,

$$10I_2 + 20I_2 - V_2 = 0$$

$$\therefore \quad z_{22} = \left|\frac{V_2}{I_2}\right|_{I_1=0} = 30\ \Omega$$

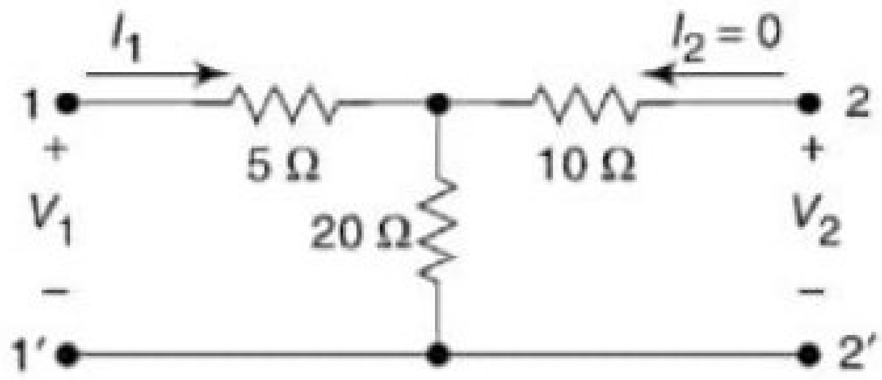
From Fig. (a), we get,

$$V_1 = 20I_2$$

$$\therefore \qquad z_{12} = \left|\frac{V_1}{I_2}\right|_{I_1=0} = 20\ \Omega$$

(b) When $I_2 = 0$, i.e., port-2 is open-circuited: In this case no current will flow through the 10 Ω resistor.

By KVL in the left mesh, we get,



$$5I_1 + 20I_1 - V_1 = 0$$

$$\therefore \qquad z_{11} = \left|\frac{V_1}{I_1}\right|_{I_2=0} = 25\ \Omega$$

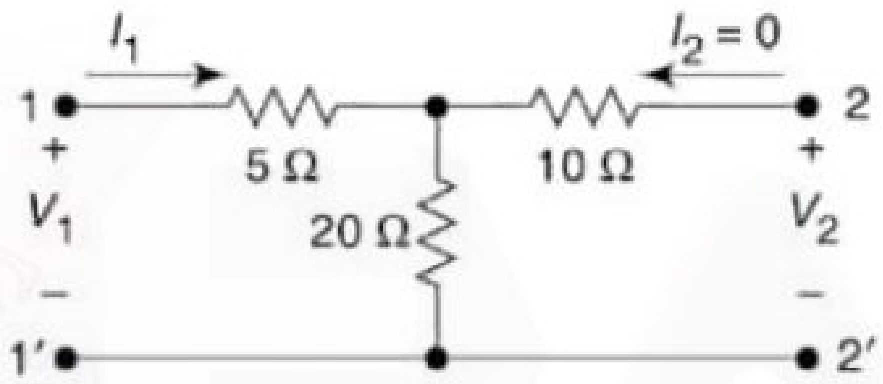


Figure (b) *When $I_2 = 0$*

From Fig. (b), we get,

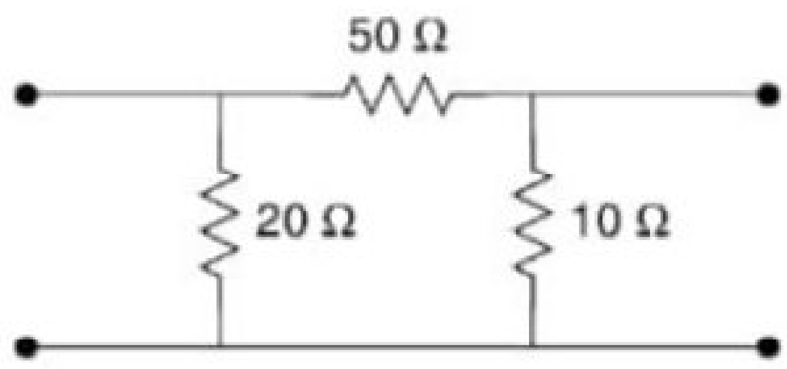
$$V_2 = 20I_1$$

$$\therefore \qquad z_{21} = \left|\frac{V_2}{I_1}\right|_{I_2=0} = 20\ \Omega$$

Therefore, the *z*-parameters of the network are:

$$[z] = \begin{bmatrix} 25 & 20 \\ 20 & 30 \end{bmatrix} (\Omega) \qquad \textit{Ans.}$$

7.29 Find the *y*-parameters for the network shown in the figure.



Solution We consider two situations:

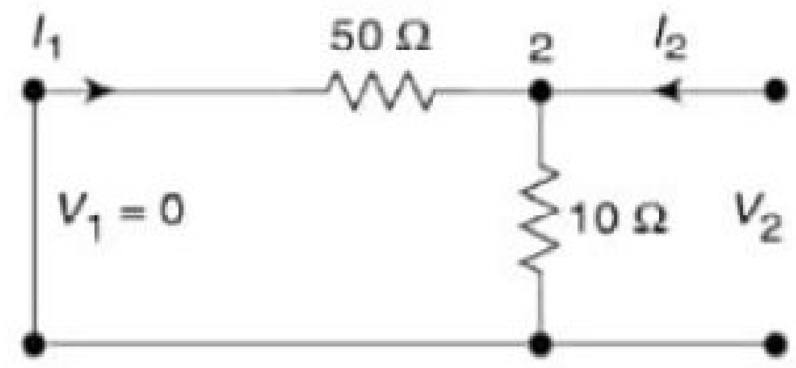
When $V_1 = 0$, i.e., port-1 is short-circuited

In this case, no current will flow through the 20 Ω resistor. The modified circuit is shown in Fig. (a).

By KCL at node 2,

$$\frac{V_2 - 0}{10} + \frac{V_2 - 0}{50} = I_2$$

$$\therefore \qquad y_{22} = \left|\frac{I_2}{V_2}\right|_{V_1=0} = \frac{1}{10} + \frac{1}{50} = 0.12\ ℧ \qquad \textit{Ans.}$$



Figure(a) *When $V_1 = 0$*

Also, from Fig. 7.5 (a) we get,

$$I_1 = \frac{0 - V_2}{50}$$

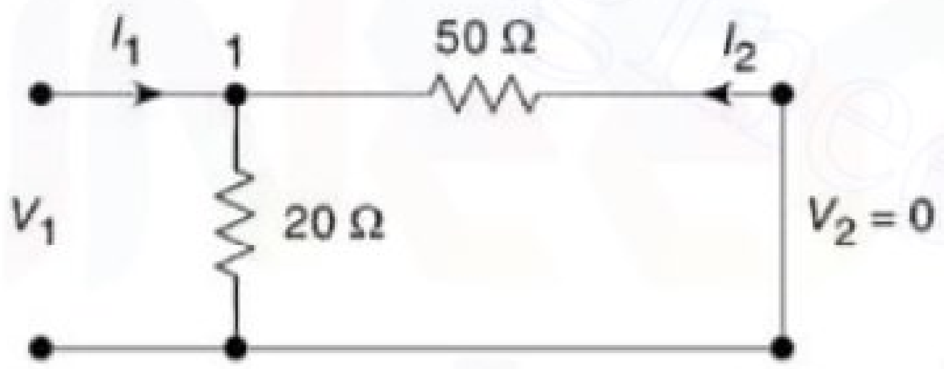
$$\therefore \quad y_{12} = \left|\frac{I_1}{V_2}\right|_{V_1=0} = \frac{1}{50} = 0.02\ \mho \qquad \textit{Ans.}$$

When $V_2 = 0$, i.e., port-2 is short-circuited

In this case, no current will flow through the 10 Ω resistor. The modified circuit is shown in Fig. (b). By KCL at node 1,

$$\frac{V_1 - 0}{20} + \frac{V_1 - 0}{50} = I_1$$

$$\therefore \quad y_{11} = \left|\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{20} + \frac{1}{50} = 0.07\ \mho \qquad \textit{Ans.}$$



Figure(b) *When $V_2 = 0$*

Also, from Fig. 7.5 (b) we get,

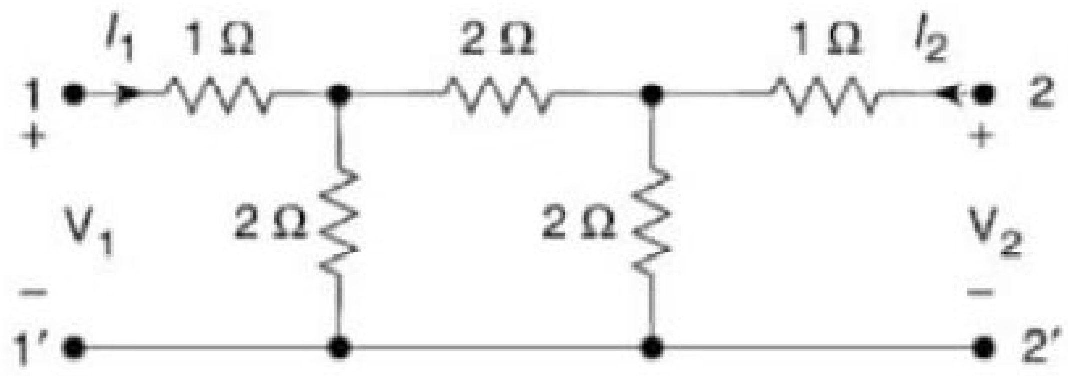
$$I_2 = \frac{0 - V_1}{50}$$

$$\therefore \quad y_{21} = \left|\frac{I_2}{V_1}\right|_{V_2=0} = \frac{1}{50} = 0.02\ \mho \qquad \textit{Ans.}$$

Therefore, the y-parameters of the network are

$$[y] = \begin{bmatrix} 0.07 & 0.02 \\ 0.02 & 0.12 \end{bmatrix} \mho \qquad \textit{Ans.}$$

7.30 For the network shown in the figure, determine the *ABCD* parameters.



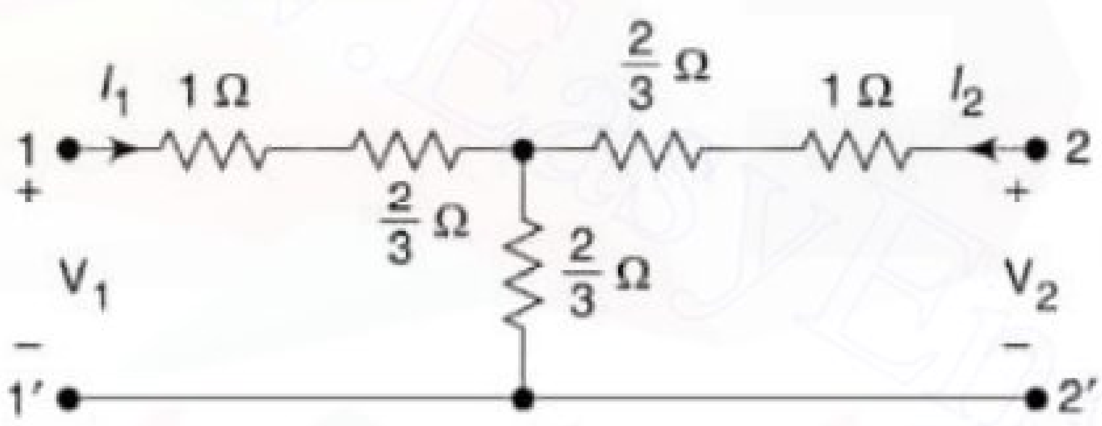
Solution The *ABCD*-parameter equations are,

$$V_1 = AV_2 - BI_2$$

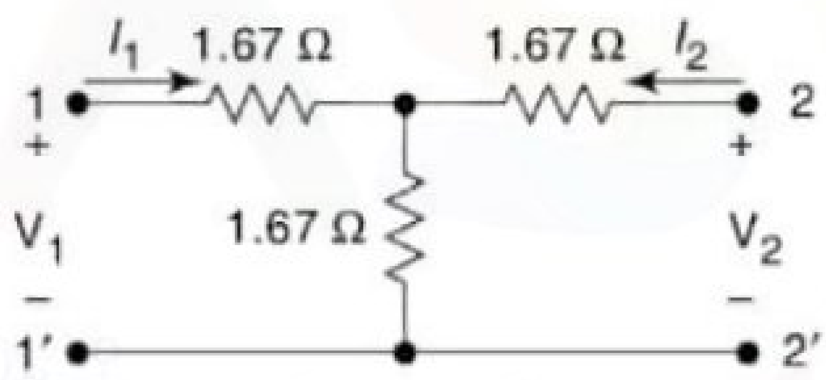
$$I_1 = CV_2 - DI_2$$

For the network shown in the figure. we convert the delta consisting of the resistances of 2 Ω each into its equivalent star so that the circuit becomes as shown in Fig. (a) and Fig. (b).

$$r_1 = r_2 = r_3 = \frac{2\times 2}{2+2+2} = \frac{2}{3}\Omega$$



(a) Modified network



(b)

To find the *ABCD* parameters, we consider two situations:

When $V_2 = 0$, i.e., port-2 is short-circuited

As shown in Fig. (c), by KVL we get,

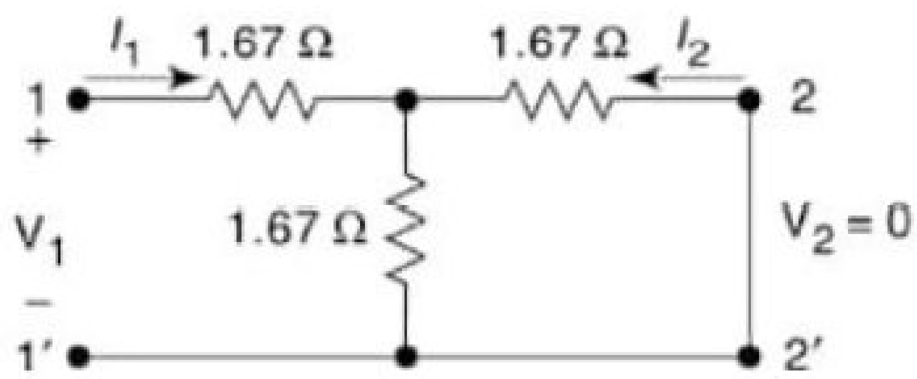
$$1.67I_1 + 0.67(I_1 + I_2) = V_1$$

or, $$2.33I_1 + 0.67I_2 = V_1$$

and, $$0.67(I_1 + I_2) + 1.67I_2 = 0$$

or, $$I_1 = -\frac{2.33}{0.67}I_2 = -3.5I_2$$

$\therefore$ $$D = \left|-\frac{I_1}{I_2}\right|_{V_2=0} = 3.5$$



(c)

Putting this value in the first equations, we get,

$$2.33\times(-3.5)I_2+0.67I_2=V_1 \Rightarrow B=\left|-\frac{V_1}{I_2}\right|_{V_2=0}=7.5\,\Omega$$

When $I_2 = 0$, i. e. port-2 is open-circuited

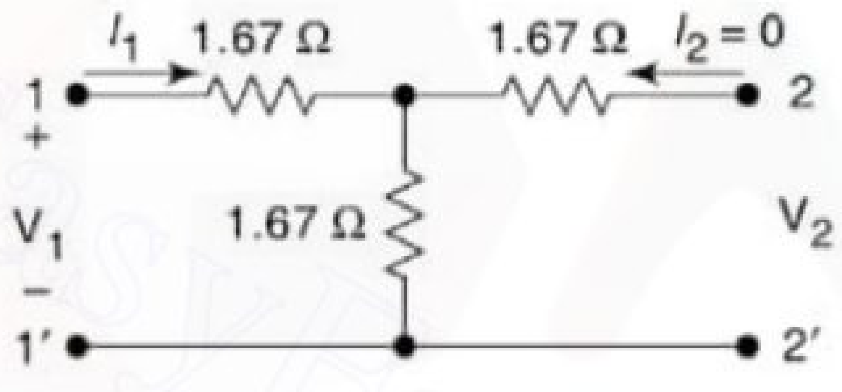
Here, no current will flow through the right side 1.67 Ω resistance. By KVL, we get,

$$V_1=(1.67+0.67)I_1=2.33I_1$$

and,
$$V_2=0.67I_1$$

∴
$$C=\left|\frac{I_1}{V_2}\right|_{I_2=0}=\frac{1}{0.67}=1.5\ \mho$$

∴
$$A=\left|\frac{V_1}{V_2}\right|_{I_2=0}=\frac{2.33I_1}{0.67I_1}=3.5$$

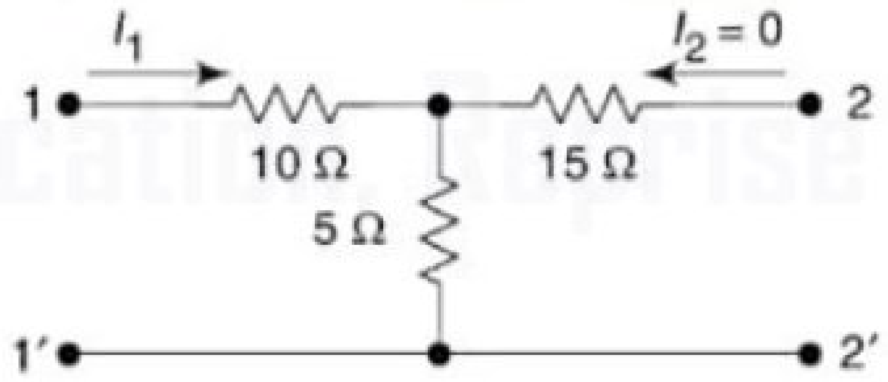


(d)

Therefore, the *ABCD* parameters of the network are

$A = 3.5$; $B = 7.5\,\Omega$; $C = 15\,\mho$; and $D = 3.5$ *Ans.*

7.31 Find the hybrid parameters for the network shown in the figure.



Solution By KVL,

$$15I_1+5I_2=V_1 \quad \text{(i)}$$

$$5I_1+20I_2=V_2 \quad \text{(ii)}$$

Thus, the z-parameters are

$z_{11}=(5+j10)\,\Omega$ $z_{12}=z_{21}=5\,\Omega$ $Z_{22}=(5+j15)\,\Omega$ *Ans.*

The hybrid parameter equations are,

$$V_1=h_{11}\,I_1+h_{12}\,V_2$$

$$I_2=h_{21}\,I_1+h_{22}\,V_2$$

From Eq (ii), we get,

$$I_2 = -\frac{5}{20} I_1 + \frac{V_2}{20}$$

$$= -\frac{1}{4} I_1 + \frac{1}{20} V_2 \qquad \text{(iii)}$$

Putting this value of I_2 in Eq (i), we get,

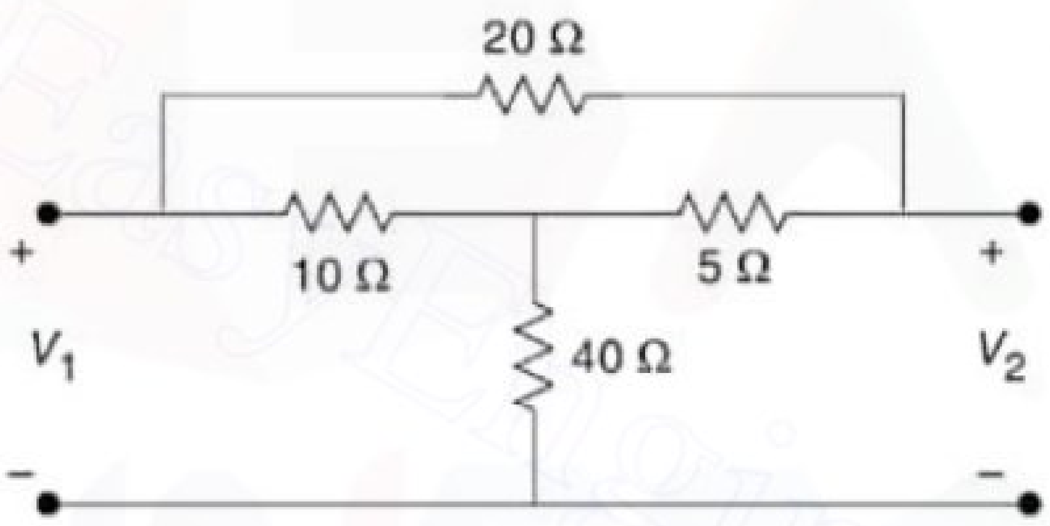
$$15I_1 + 5\left[-\frac{1}{4} I_1 + \frac{V_2}{20}\right] = V_1$$

$$\Rightarrow \qquad V_1 = \frac{55}{4} I_1 + \frac{1}{4} V_2 \qquad \text{(iv)}$$

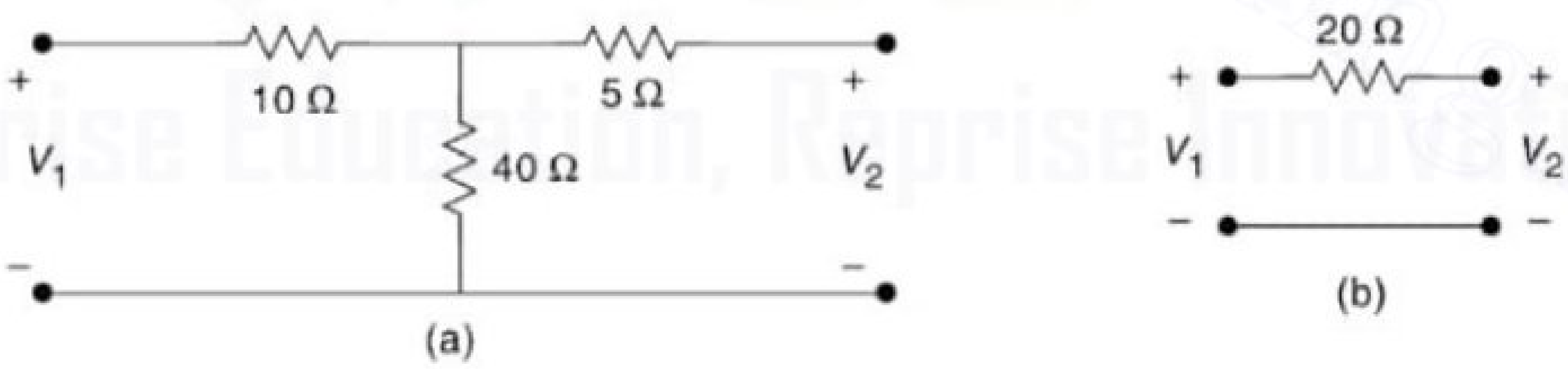
Comparing Eq (iii) and (iv) with the standard equations of h-parameters, we get,

$$h_{11} = \frac{55}{4}\,\Omega; \quad h_{12} = \frac{1}{4}; \quad h_{21} = -\frac{1}{4}; \quad h_{22} = \frac{1}{20}\,\mho \qquad \textit{Ans.}$$

7.32 Find the y parameters for the following network:



Solution This two-port network can be considered as the parallel connection of two two-port networks as shown below.



For network (a), the z-parameters are:

$$z_{11a} = 50\,\Omega; \quad z_{12a} = z_{21a} = 40\,\Omega; \quad z_{22a} = 45\,\Omega; \qquad \therefore \Delta z = \left(50 \times 45 - 40^2\right) = 650$$

Thus, the y-parameters are

$$y_{11a} = \frac{z_{22a}}{\Delta z} = \frac{45}{650} = \frac{9}{130} \text{ mho}$$

$$y_{12a} = y_{21a} = -\frac{z_{12}}{\Delta z} = -\frac{40}{650} = -\frac{4}{65} \text{ mho}$$

$$y_{22a} = \frac{z_{11a}}{\Delta z} = \frac{50}{650} = \frac{1}{13} \text{ mho}$$

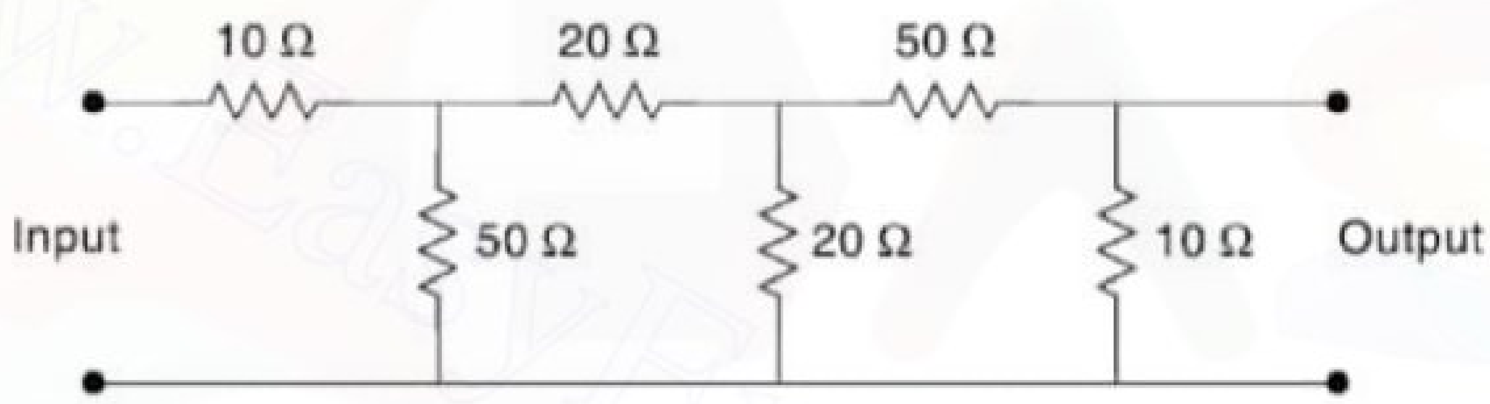
For network (b), the y-parameters are

$$y_{11b} = y_{22b} = \frac{1}{20} \text{ mho}; \quad y_{12b} = y_{21b} = -\frac{1}{20} \text{ mho}$$

We know that for parallel connection of two two-port networks the overall y-parameters are the summation of individual y-parameters. Thus,

$$\left.\begin{aligned} y_{11} &= (y_{11a} + y_{11b}) = \left(\frac{9}{130} + \frac{1}{20}\right) = 0.119 \text{ mho} \\ y_{12} &= y_{21} = (y_{12a} + y_{12b}) = \left(-\frac{4}{65} - \frac{1}{20}\right) = -0.111 \text{ mho} \\ y_{22} &= (y_{22a} + y_{22b}) = \left(\frac{1}{13} + \frac{1}{20}\right) = 0.127 \text{ mho} \end{aligned}\right\} \quad \textit{Ans.}$$

7.33 Obtain the *ABCD* parameters for the network shown in the figure.



Solution This two-port network can be considered as the cascade connection of two two-port networks as shown below.

For Network (a), as this is a T-network, the z-parameters are given as,

$$z_{11} = 60\ \Omega; \quad z_{12} = 50\ \Omega; \quad z_{22} = 70\ \Omega; \qquad \therefore \Delta z = (z_{11}z_{22} - z_{12}z_{21}) = (60 \times 70 - 50^2) = 1700$$

$$\therefore \quad A_a = \frac{z_{11}}{z_{21}} = \frac{60}{50} = \frac{6}{5} \qquad B_a = \frac{\Delta z}{z_{21}} = \frac{1700}{50} = 34\ \Omega$$

$$C_a = \frac{1}{z_{21}} = \frac{1}{50} \text{ mho} \qquad D_a = \frac{z_{22}}{z_{21}} = \frac{70}{50} = \frac{7}{5}$$

For Network (b), as this is a π-network, the y-parameters are given as,

$$y_{11} = \left(\frac{1}{50} + \frac{1}{20}\right) = \frac{7}{100} \text{ mho}; \quad y_{12} = y_{21} = -\frac{1}{50} \text{ mho}; \quad y_{22} = \left(\frac{1}{50} + \frac{1}{10}\right) = \frac{3}{25} \text{ mho}$$

$$\therefore \quad \Delta y = (y_{11}y_{22} - y_{12}y_{21}) = \frac{7}{100} \times \frac{3}{25} - \left(-\frac{1}{50}\right)^2 = \frac{1}{125}$$

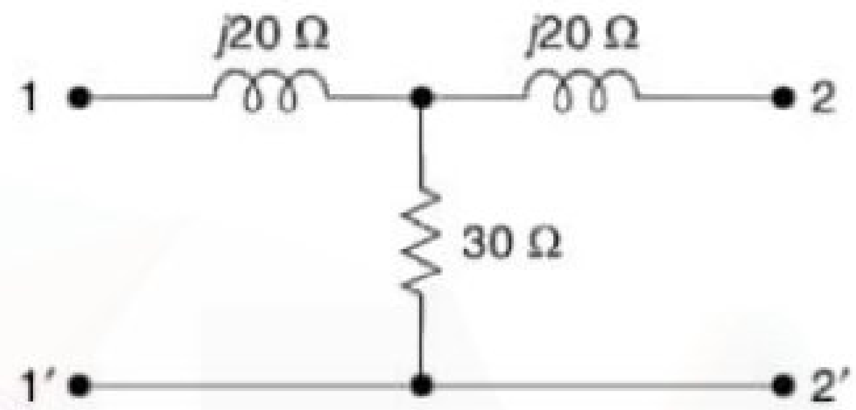
$$\therefore \quad A_b = -\frac{y_{22}}{y_{21}} = -\frac{3/25}{-1/50} = 6 \qquad B_b = -\frac{1}{y_{21}} = -\frac{1}{-1/50} = 50\ \Omega$$

$$C_b = -\frac{\Delta y}{y_{21}} = -\frac{1/125}{-1/50} = \frac{2}{5}\text{ mho} \qquad D_b = -\frac{y_{11}}{y_{21}} = -\frac{7/100}{-1/50} = \frac{7}{2}$$

For the entire network, the *ABCD* parameters are given as,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \times \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} = \begin{bmatrix} 6/5 & 34 \\ 1/50 & 7/5 \end{bmatrix} \times \begin{bmatrix} 6 & 50 \\ 2/5 & 7/5 \end{bmatrix} = \begin{bmatrix} 20.8 & 179 \\ 0.68 & 5.9 \end{bmatrix} \qquad \textit{Ans.}$$

7.34 Calculate the *ABCD* parameters of the network shown in the figure below.



Solution For this *T*-circuit, the *z*-parameters are given as,

$$z_{11} = z_{22} = (30 + j20)\ \Omega$$

$$z_{12} = z_{21} = 30\ \Omega$$

$$\therefore \quad \Delta z = (z_{11}z_{22} - z_{12}\,z_{21}) = (30 + j20)^2 - 30^2 = (60 + j20)j20 = (-400 + j1200)$$

$$\therefore \quad A = \frac{z_{11}}{\Delta z} = \frac{30 + j20}{(60 + j20)j20} = \left(1 + j\frac{2}{3}\right)$$

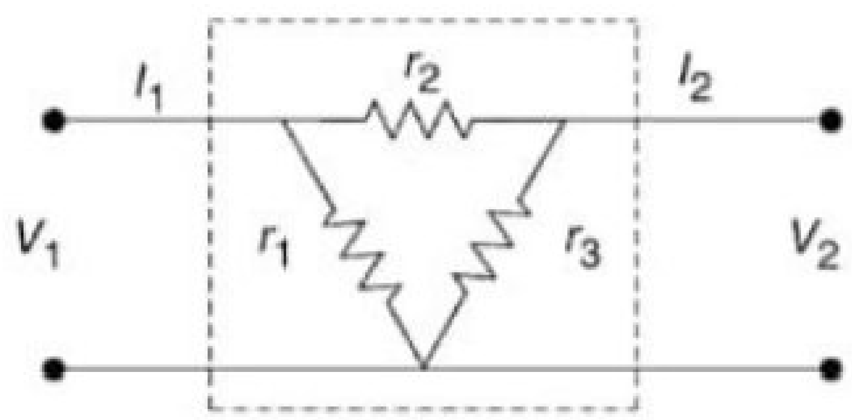
$$\therefore \quad B = \frac{\Delta z}{z_{21}} = \frac{(60 + j20)j20}{30} = \left(-\frac{40}{3} + j40\right)\Omega$$

$$\therefore \quad C = \frac{1}{z_{12}} = \frac{1}{30}\ mho$$

$$\therefore \quad D = \frac{z_{22}}{z_{12}} = \frac{30 + j20}{30} = \left(1 + j\frac{2}{3}\right)$$

Ans.

7.35 Determine the hybrid parameters for the network in the figure shown below.



Solution For this π-network, the *y*-parameters are given as,

$$y_{11} = \left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \left(\frac{r_1 + r_2}{r_1 r_2}\right); \quad y_{12} = y_{21} = -\frac{1}{r_2}; \qquad y_{22} = \left(\frac{1}{r_2} + \frac{1}{r_3}\right) = \left(\frac{r_2 + r_3}{r_2 r_3}\right)$$

By inter-relationship, the h-parameters are obtained as,

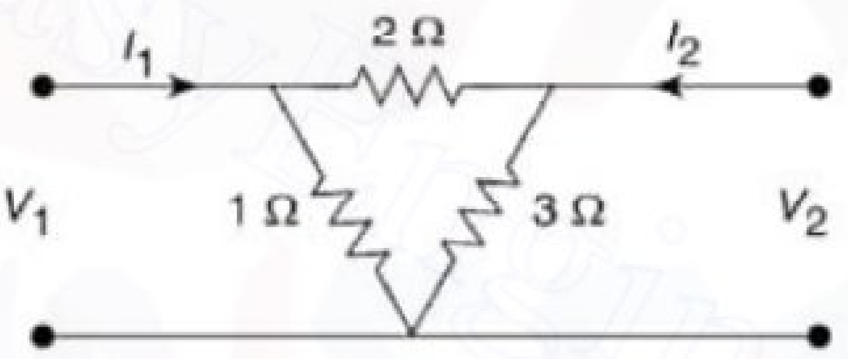
$$h_{11} = \frac{1}{y_{11}} = \left(\frac{r_1 r_2}{r_1 + r_2}\right)$$

$$h_{12} = -\frac{y_{12}}{y_{11}} = -\frac{-\dfrac{1}{r_2}}{\left(\dfrac{r_1 + r_2}{r_1 r_2}\right)} = \frac{r_1}{r_1 + r_2}$$

$$h_{21} = \frac{y_{21}}{y_{11}} = \frac{-\dfrac{1}{r_2}}{\left(\dfrac{r_1 + r_2}{r_1 r_2}\right)} = -\frac{r_1}{r_1 + r_2}$$

$$h_{22} = \frac{\Delta y}{y_{11}} = \left\{\frac{(r_1 + r_2)(r_2 + r_3) - r_1 r_3}{r_1 r_2^2 r_3}\right\} \times \left(\frac{r_1 r_2}{r_1 + r_2}\right) = \frac{(r_1 + r_2)(r_2 + r_3) - r_1 r_3}{r_2(r_1 + r_2)}$$

7.36 Find the hybrid parameters of the circuit given in the figure.



Solution For this π-network, the y-parameters are given as,

$$y_{11} = \left(\frac{1}{1} + \frac{1}{2}\right) = \frac{3}{2}; \quad y_{12} = y_{21} = -\frac{1}{2}; \qquad y_{22} = \left(\frac{1}{2} + \frac{1}{3}\right) = \frac{5}{6}$$

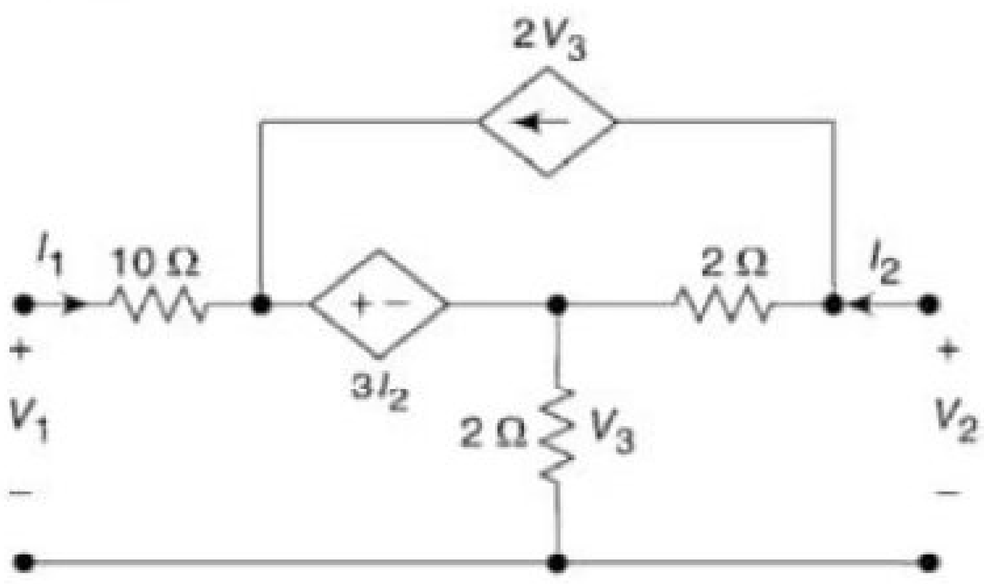
$$\therefore \qquad \Delta y = y_{11}\, y_{22} - y_{12}\, y_{21} = \frac{3}{2} \times \frac{5}{6} - \left(-\frac{1}{2}\right)^2 = 1$$

By inter-relationship, the h-parameters are obtained as,

$$h_{11} = \frac{1}{y_{11}} = \frac{2}{3}\,\Omega \qquad\qquad h_{12} = -\frac{y_{12}}{y_{11}} = -\frac{-1/2}{3/2} = \frac{1}{3}$$

$$h_{21} = \frac{y_{21}}{y_{11}} = \frac{-\dfrac{1}{2}}{3/2} = -\frac{1}{3} \qquad\qquad h_{22} = \frac{\Delta y}{y_{11}} = 1 \times \frac{3}{2} = \frac{3}{2}\ \mho$$

7.37 For the network shown in the figure, determine the z and y parameters.



Solution By KVL for the three meshes, we get,

$V_1 = 10I_1 + 3I_2 + 2(I_1 + I_2) \Rightarrow 12I_1 + 5I_2 = V_1$ (i)

$V_2 = 2(I_2 - 2V_3) + 2(I_1 + I_2) \Rightarrow 2I_1 + 4I_2 - 4V_3 = V_2$ (ii)

$V_3 = 2(I_1 + I_2)$ (iii)

From (ii) and (iii),

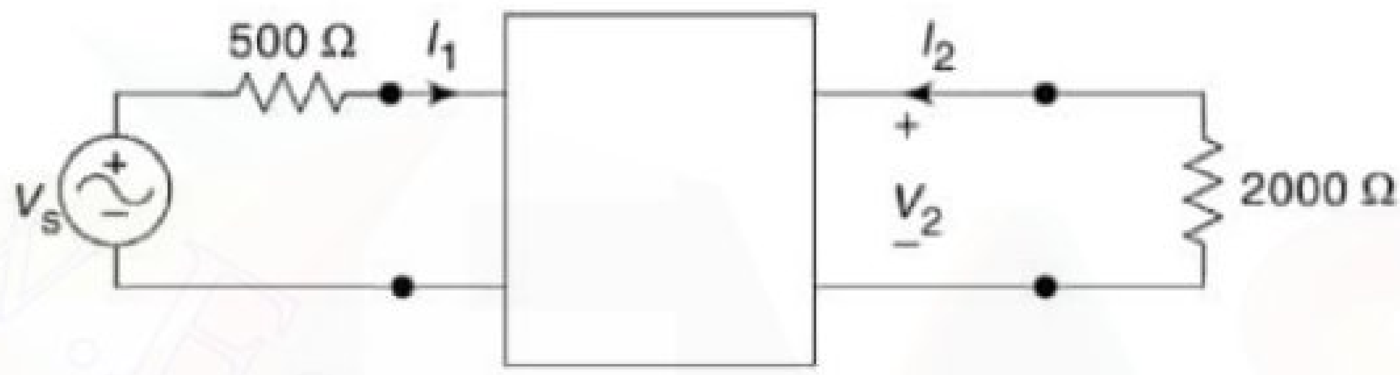
$$V_2 = 2I_1 + 4I_2 - 4(2I_1 + 2I_2) \Rightarrow V_2 = -6I_1 - 4I_2 \quad \text{(iv)}$$

From (i) and (iv), we get,

$$z = \begin{bmatrix} 12 & 5 \\ -6 & -4 \end{bmatrix} (\Omega) \qquad \textit{Ans.}$$

$$\therefore \qquad y = [z]^{-1} = \begin{bmatrix} 2/9 & 5/18 \\ -1/3 & -2/3 \end{bmatrix} (\mho) \qquad \textit{Ans.}$$

7.38 The h-parameters of a two-port network shown in figure are $h_{11} = 1000\Omega$, $h_{12} = 0.003$, $h_{21} = 100$, and $h_{22} = 50 \times 10^{-6}$ mho. Find V_2 and z-parameters of the network if $V_s = 10^{-2}\angle 0^\circ\ (V)$.



Solution The h-parameter equations are,

$$V_1 = h_{11}I_1 + h_{12}V_2 = 1000I_1 + 0.003V_2 \quad \text{(i)}$$

$$I_2 = h_{21}I_1 + h_{22}V_2 = 100I_1 + 50 \times 10^{-6}V_2 \quad \text{(ii)}$$

By KVL for the two meshes,

$$V_1 = V_s - 500I_1 \quad \text{(iii)}$$

$$V_2 = -200I_2 \quad \text{(iv)}$$

From (i) and (iii),

$$V_s - 500I_1 = 1000I_1 + 0.003V_2$$

or,

$$10^{-2} - 1500I_1 = 0.003V_2 \quad \text{(v)}$$

From (ii) and (iv),

$$-\frac{V_2}{2000} = 100I_1 + 50 \times 10^{-6} V_2$$

or,

$$I_1 = -5.5 \times 10^{-6} V_2 \quad \text{(vi)}$$

From (v) and (i),

$$0.003V_2 = 10^{-2} + 1500(-5.5 \times 10^{-6} V_2)$$

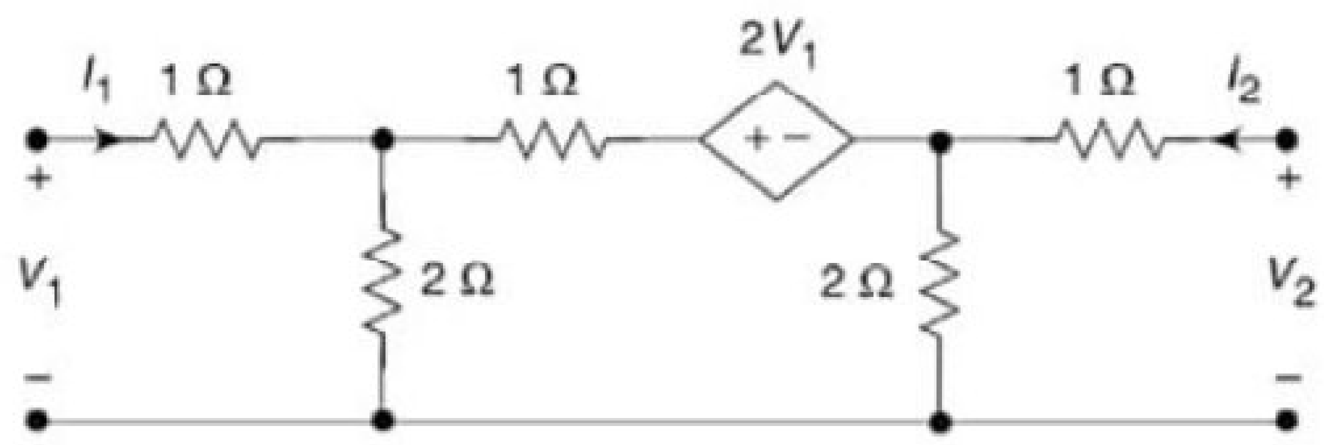
$$\Rightarrow \qquad V_2 = -1.905\ V \qquad \textit{Ans.}$$

The z-parameters are calculated as follows.

$$z_{11} = \frac{\Delta h}{h_{22}} = -500\Omega \quad z_{12} = \frac{h_{12}}{h_{22}} = 60\Omega \quad z_{21} = -\frac{h_{21}}{h_{22}} = -2 \times 10^6\,\Omega$$

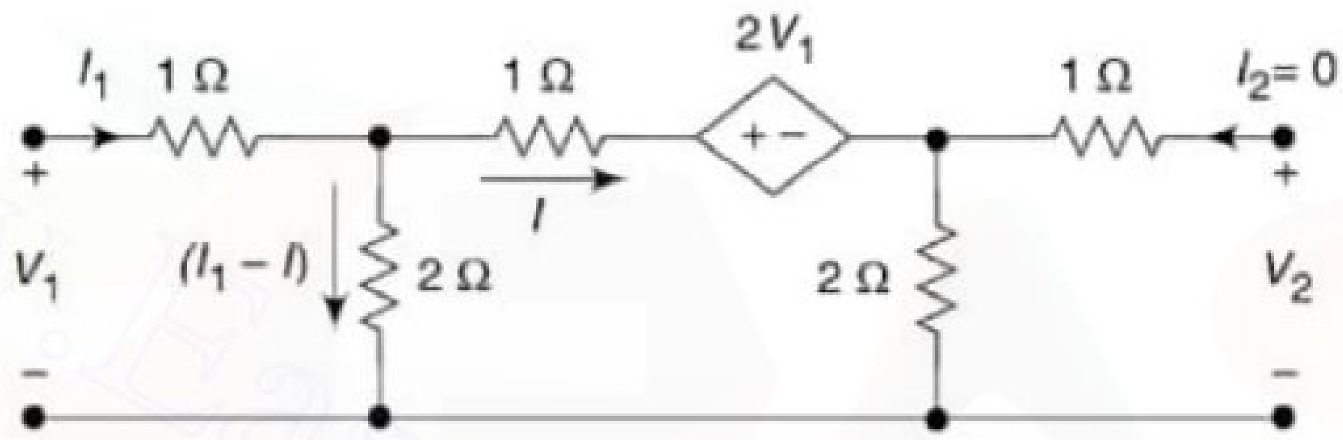
$$z_{22} = \frac{1}{h_{22}} = 20 \times 10^3\,\Omega \qquad \textit{Ans.}$$

7.39 For the two-port network shown in the figure, find the z-parameters.



Solution We consider two cases:

When $I_2 = 0$ Here, as the output port is open-circuited, no current will flow through the 1Ω resistor connected at port 2. The modified circuit is shown in Fig (a).



(a)

By KVL for the middle mesh, we get,

$$I + 2V_1 + 2I - 2 \times (I_1 - I) = 0$$

$$\Rightarrow \qquad I = \left(\frac{2}{5}I_1 - \frac{2}{5}V_1\right) \qquad \text{(i)}$$

By KVL for the left mesh, we get,

$$V_1 = I_1 + 2 \times (I_1 - I) = 3I_1 - 2I$$

$$= 3I_1 - 2 \times \left(\frac{2}{5}I_1 - \frac{2}{5}V_1\right) \qquad \{\text{by equation (i)}\}$$

or, $$V_1 = 11I_1$$

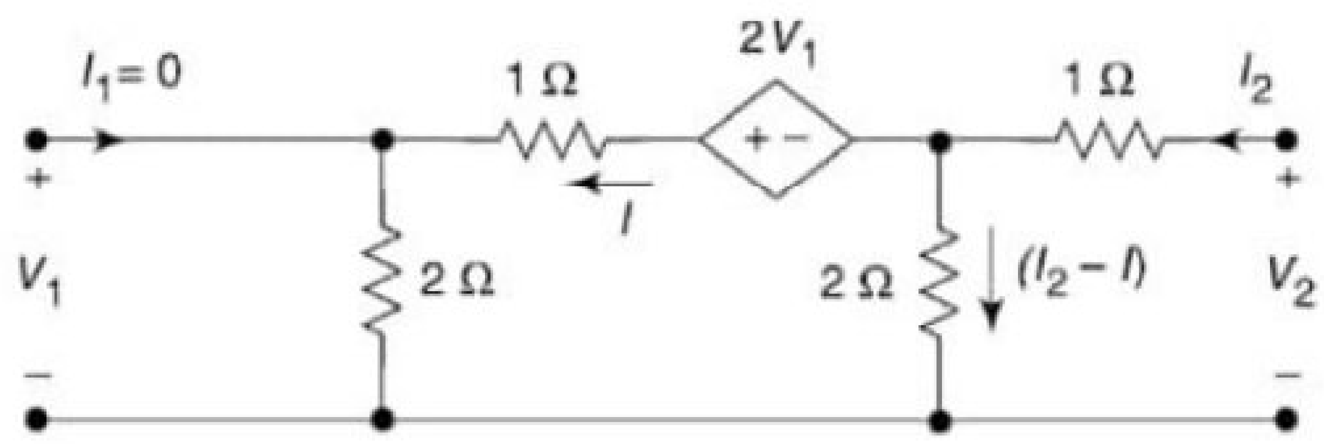
$$\therefore \qquad z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 11\ \Omega$$

Also, by KVL for the right mesh, we get,

$$V_2 = 2I = 2 \times \left(\frac{2}{5}I_1 - \frac{2}{5}V_1\right) = \frac{4}{5}I_1 - \frac{4}{5}V_1 = \frac{4}{5}I_1 - \frac{4}{5} \times 11 \times I_1 = -8I_1$$

$$\therefore \qquad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = -8\ \Omega$$

When $I_1 = 0$ Here, as the output port is open-circuited, no current will flow through the 1 Ω resistor connected at port 1. The modified circuit is shown in Fig (b).



(b)

By KVL for the middle mesh, we get,

$$I - 2V_1 + 2I - 2 \times (I_2 - I) = 0$$

$$\Rightarrow \qquad I = \left(\frac{2}{5}I_2 + \frac{2}{5}V_1\right) \qquad \text{(ii)}$$

By KVL for the left mesh, we get,

$$V_1 = 2I = 2\times\left(\frac{2}{5}I_2 + \frac{2}{5}V_1\right) = \frac{4}{5}I_2 + \frac{4}{5}V_1$$

$$\Rightarrow \qquad V_1 = 4I_2$$

$$\therefore \qquad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=} = 4\ \Omega$$

Also, by KVL for the right mesh, we get,

$$V_2 = I_2 + 2\times(I_2 - I) = 3I_2 - 2I$$

$$= 3I_2 - 2\times\left(\frac{2}{5}I_2 + \frac{2}{5}V_1\right) \qquad \{by\ equation\ (ii)\}$$

$$= \frac{11}{5}I_2 - \frac{4}{5}V_1 = \frac{11}{5}I_2 - \frac{4}{5}\times 4I_2 = -I_2$$

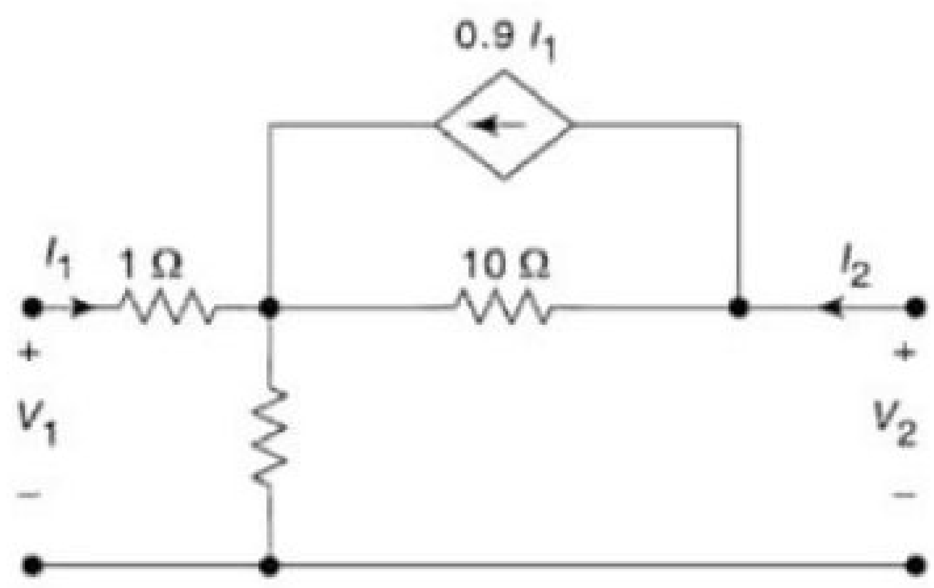
$$\therefore \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = -1\ \Omega$$

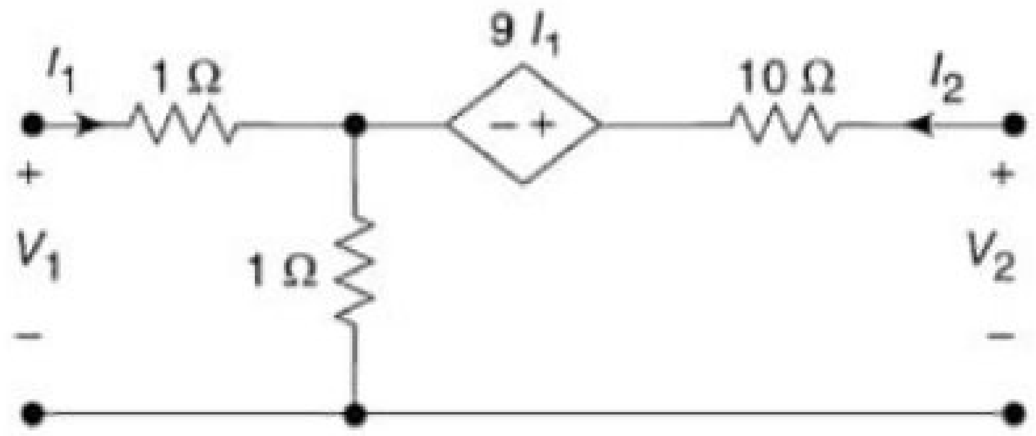
Therefore, the z-parameters of the network are,

$$[z] = \begin{bmatrix} 11 & 4 \\ -8 & -1 \end{bmatrix}(\Omega) \qquad \textit{Ans.}$$

7.40 Find the z and y parameters of the network shown in the figure.

Solution We convert the dependent current source into its equivalent voltage source as shown in the figure below.





By KVL for the two meshes, we get,

$$I_1 + 1 \times (I_1 + I_2) = V_1 \Rightarrow V_1 = 2I_1 + I_2 \qquad \text{(i)}$$

and,

$$10I_2 + 9I_1 + 1 \times (I_1 + I_2) = V_2 \Rightarrow V_2 = 10I_1 + 11I_2 \qquad \text{(ii)}$$

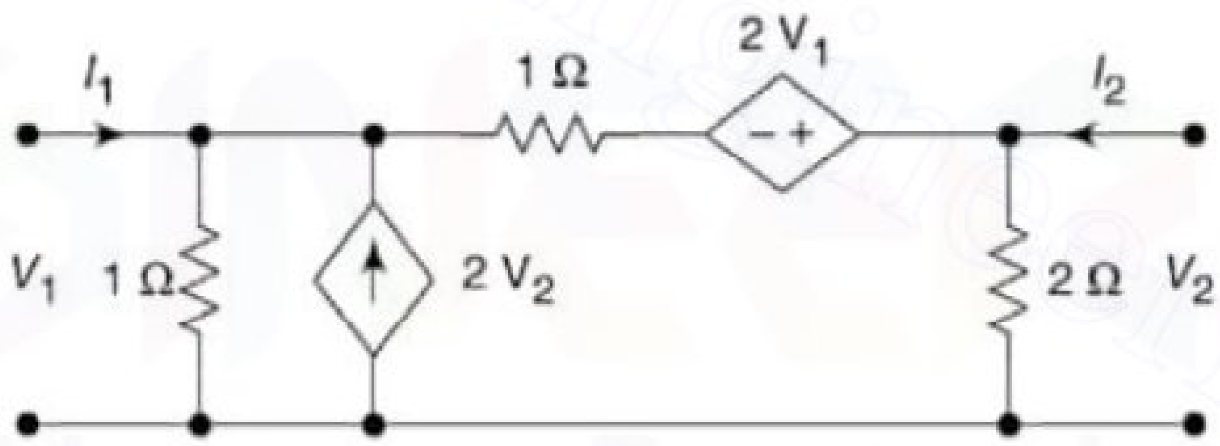
From (i) and (ii), we get the z-parameters as,

$$[z] = \begin{bmatrix} 2 & 1 \\ 10 & 11 \end{bmatrix} (\Omega) \qquad \textit{Ans.}$$

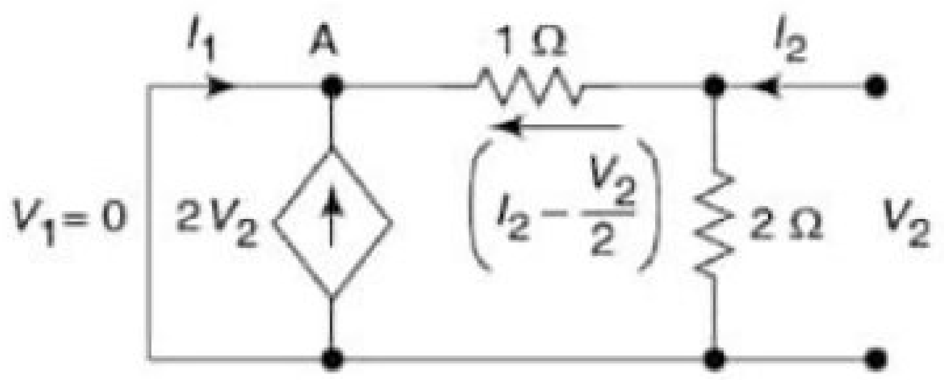
Therefore, the y-parameters are,

$$[y] = [z]^{-1} = \begin{bmatrix} 2 & 1 \\ 10 & 11 \end{bmatrix}^{-1} = \begin{bmatrix} 11/12 & -1/12 \\ -10/12 & 2/12 \end{bmatrix} \mho \qquad \textit{Ans.}$$

7.41 The network shown in the figure contains both dependent current source and dependent voltage source. For this circuit, determine the *y* and *z* parameters.



Solution We first find out the *y* parameters. To find the *y* parameters, we consider two situations:
When $V_1 = 0$ Here, port 1 is shorted and hence, the dependent voltage source is zero, i.e., short-circuited. The 1 Ω resistance in port 1 becomes redundant. The circuit is shown in Fig (a).



(a)

By KCL at node (*A*), we get,

$$-I_1 - 2V_2 - \left(I_2 - \frac{V_2}{2}\right) = 0 \Rightarrow I_1 + I_2 = -\frac{3V_2}{2} \qquad \text{(i)}$$

By KVL for the outer loop, we get,

$$V_2 = 1\times\left(I_2 - \frac{V_2}{2}\right) = I_2 - \frac{V_2}{2}$$

$$\Rightarrow \quad \frac{3}{2}V_2 = I_2$$

$$\therefore \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{3}{2}\ \mho$$

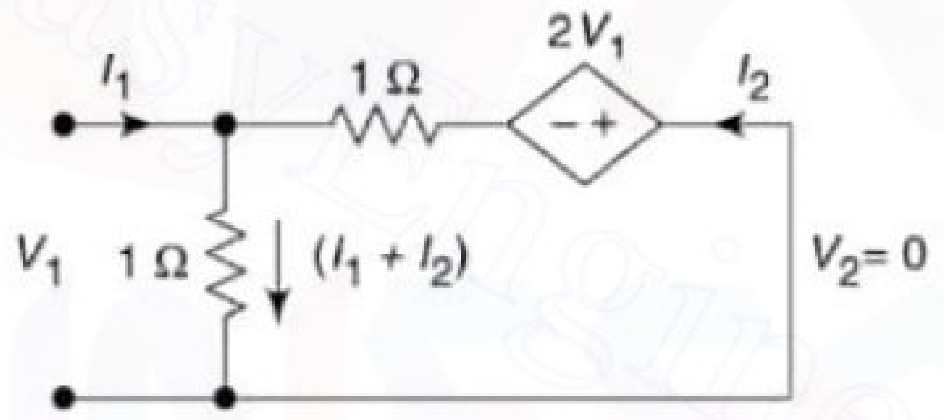
Substituting the value of I_2 in (i), we get,

$$I_1 + \frac{3}{2}V_2 = -\frac{3}{2}V_2$$

$$\Rightarrow \quad I_1 = -3V_2$$

$$\therefore \quad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -3\ \mho$$

When $V_2 = 0$ Here, port 2 is shorted and hence, the dependent current source is zero, i.e., open-circuited. The 2 Ω resistance in port 2 becomes redundant. The circuit is shown in Fig (b).



(b)

By KVL for the left loop, we get,

$$V_1 = (I_1 + I_2) \qquad \text{(ii)}$$

By KVL for the outer loop, we get,

$$2V_1 + I_2 + V_1 = 0 \quad \Rightarrow \quad I_2 = -3V_1$$

$$\therefore \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -3\ \mho$$

From (ii),

$$V_1 = I_1 - 3V_1 \quad \Rightarrow \quad I_1 = 4V_1$$

$$\therefore \quad y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 4\ \mho$$

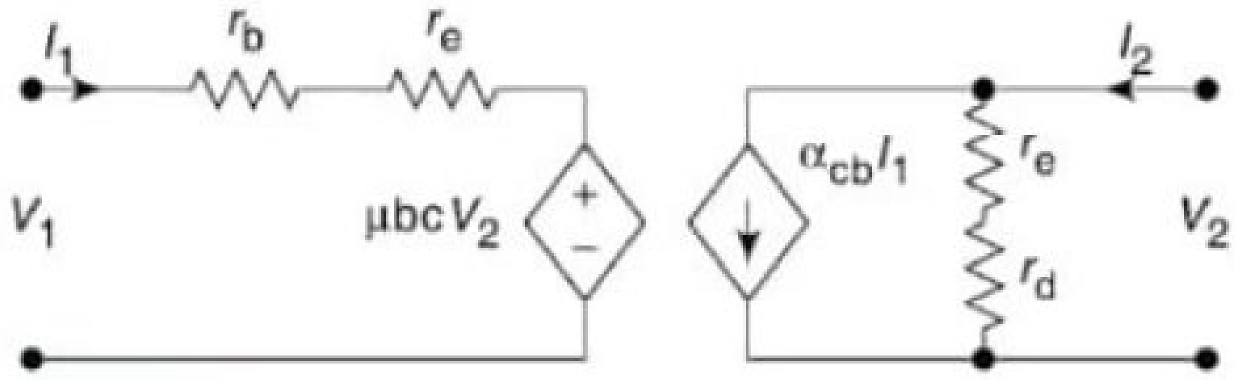
Therefore, the y parameters of the netwrok is given as,

$$[y] = \begin{bmatrix} 4 & -3 \\ -3 & \frac{3}{2} \end{bmatrix} \qquad \textit{Ans.}$$

Hence, the z parameters are given as,

$$[z]=[y]^{-1}=\begin{bmatrix} 4 & -3 \\ -3 & \frac{3}{2} \end{bmatrix}^{-1}=\begin{bmatrix} \frac{-1}{2} & -1 \\ -1 & -\frac{4}{3} \end{bmatrix}(\Omega) \qquad Ans.$$

7.42 The model of a transistor in *CE* mode is shown in the figure. Determine the *h* parameters of the model.

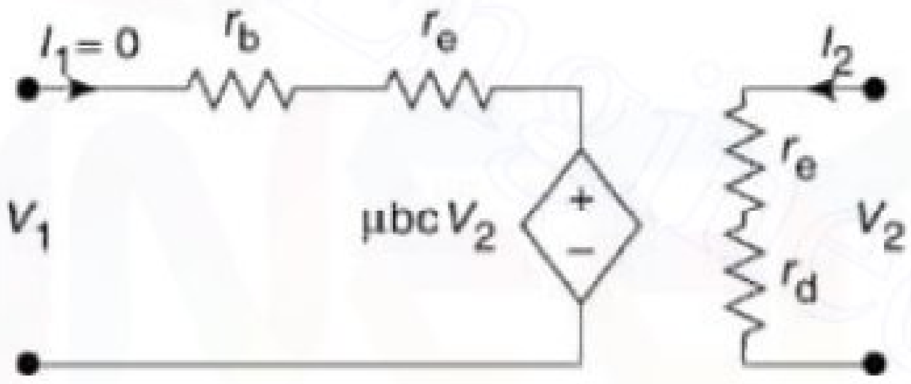


Solution The equations of *h* parameters are,

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

To find *h* parameters, we consider two cases:

When $I_1 = 0$ Here, the dependent current source is open-circuited. The modified circuit is shown in Fig (a).



(a)

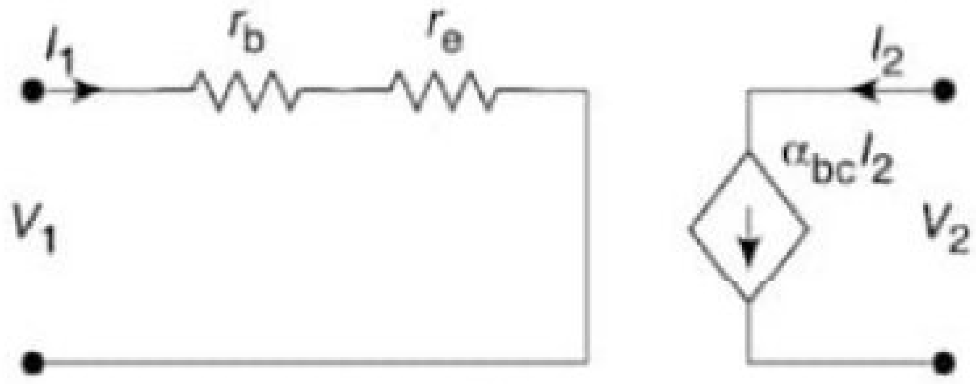
$$\therefore \qquad V_1 = \mu_{bc} V_2$$

$$\Rightarrow \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \mu_{bc}$$

Also, $V_2 = I_2 (r_e + r_d)$

$$\Rightarrow \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{r_e + r_d} \mho$$

When $V_2 = 0$ Here, the dependent voltage source is short-circuited. The modified circuit is shown in Fig (b).



(b)

$\therefore \qquad V_1 = I_1(r_b + r_e)$

$\Rightarrow \qquad h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = (r_b + r_e)\Omega$

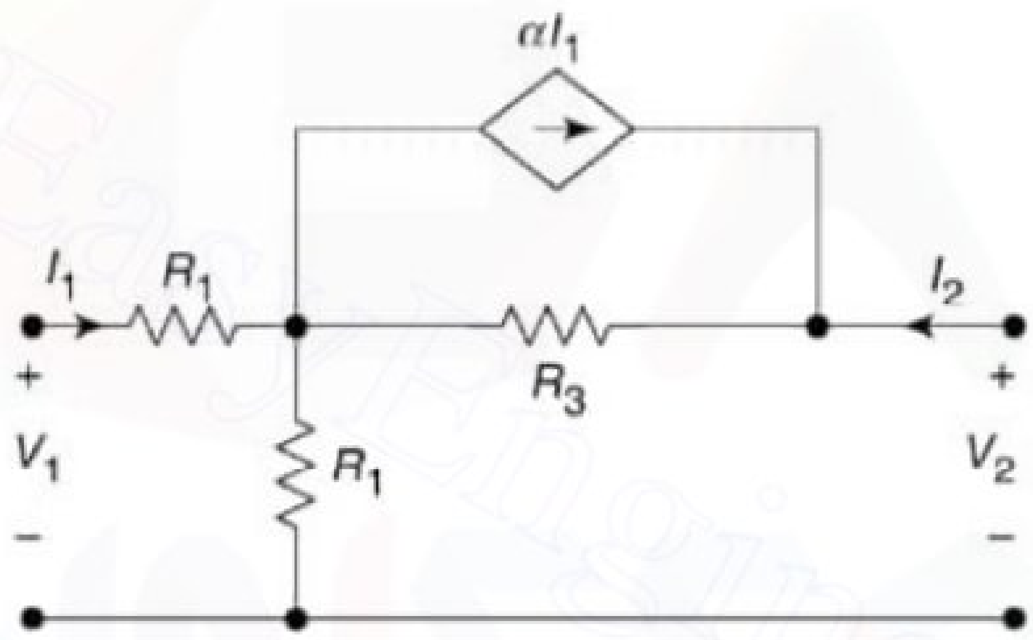
Also, $I_2 = \alpha_{cb} I_1$

$\Rightarrow \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \alpha_{cb}$

Therefore, the h parameters for the transistor model is given as,

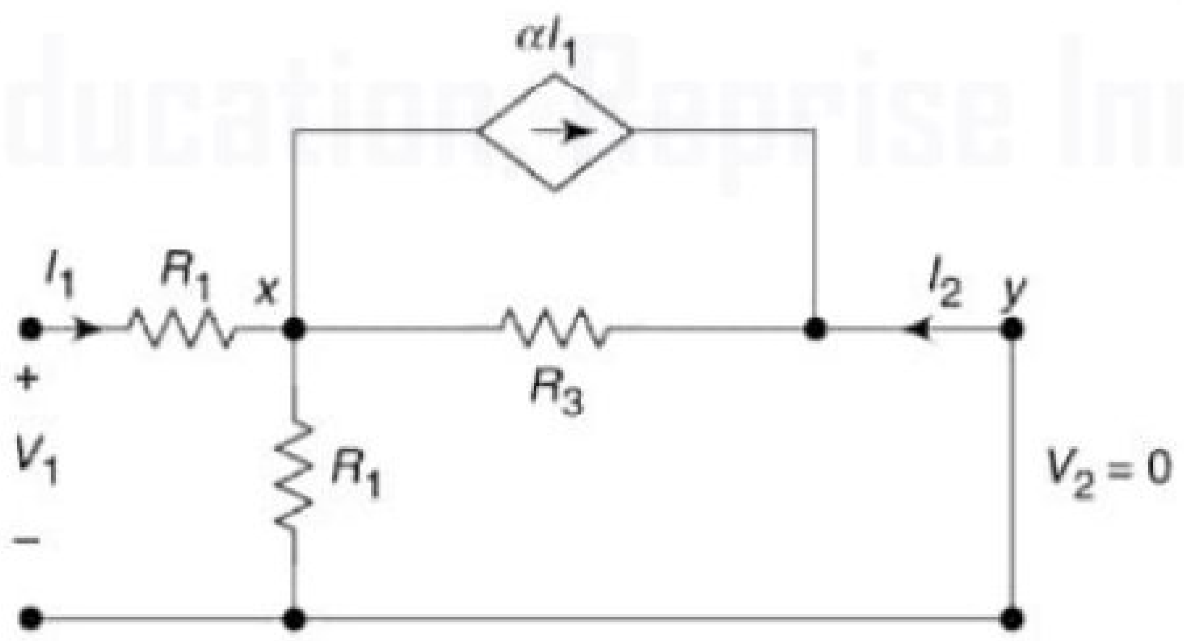
$$[h] = \begin{bmatrix} (r_b + r_e) & \mu_{bc} \\ \alpha_{cb} & \frac{1}{r_e + r_d} \end{bmatrix} \qquad Ans.$$

7.43 Find the hybrid parameters for the network of the figure (which represents a transistor).



Solution **Case (I): When $V_2 = 0$**

The circuit is modified as shown in the figure.



By KCL at node x,

$$\frac{V_x}{R_2} + \frac{V_x}{R_3} + \alpha I_1 = I_1 \qquad \Rightarrow \qquad V_x = (1-\alpha)\frac{R_2 R_3}{R_2 + R_3} I_1$$

By KVL,

$$V_1 = I_1 R_1 + V_x = I_1 R_1 + (1-\alpha)\left(\frac{R_2 R_3}{R_2 + R_3}\right) I_1$$

$$\therefore \qquad h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} = \left[R_1 + \frac{(1-\alpha)R_2 R_3}{R_2 + R_3}\right] \qquad \textit{Ans.}$$

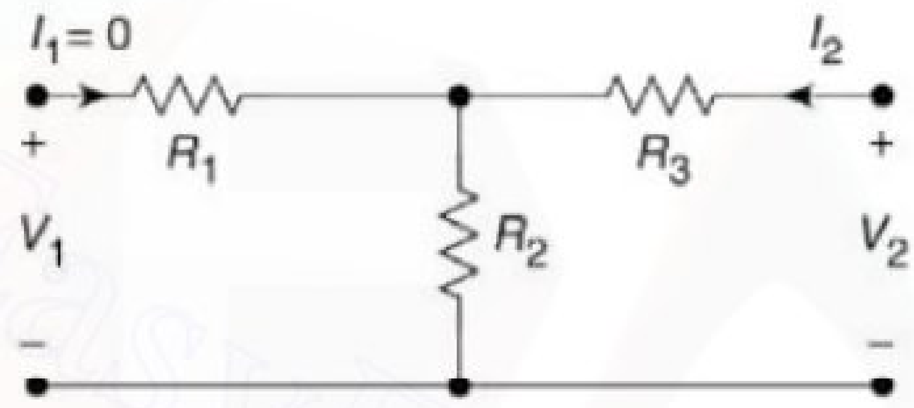
By KCL at node y,

$$\frac{0 - V_x}{R_3} = I_2 + \alpha I_1 \quad \Rightarrow \quad I_2 = -\alpha I_1 - (1-\alpha)\left(\frac{R_2 R_3}{R_2 + R_3}\right) I_1 = -I_1\left(\frac{R_2 + \alpha R_3}{R_2 + R_3}\right)$$

$$\therefore \qquad h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0} = -\left(\frac{R_2 + \alpha R_3}{R_2 + R_3}\right) \qquad \textit{Ans.}$$

Case (II): When $I_1 = 0$

Here, the dependent current source is to be opened (since $I_1 = 0$).
The circuit is modified as shown in the figure.



$$\therefore \qquad V_2 = I_2(R_2 + R_3)$$

and $\qquad V_1 = I_2 R_2$

$$\therefore \qquad h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0} = \frac{R_2}{R_2 + R_3} \qquad \textit{Ans.}$$

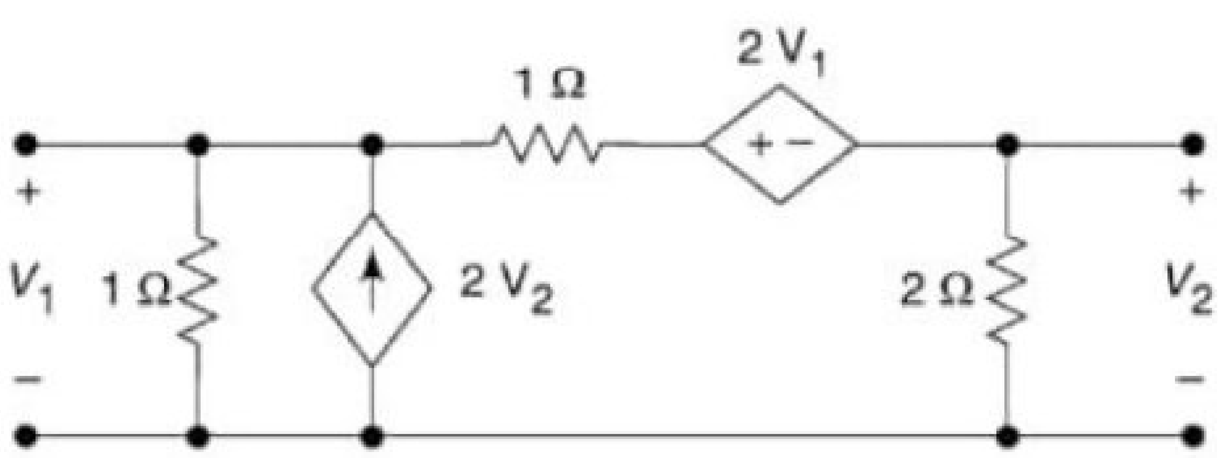
and

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} = \frac{1}{R_2 + R_3} \qquad \textit{Ans.}$$

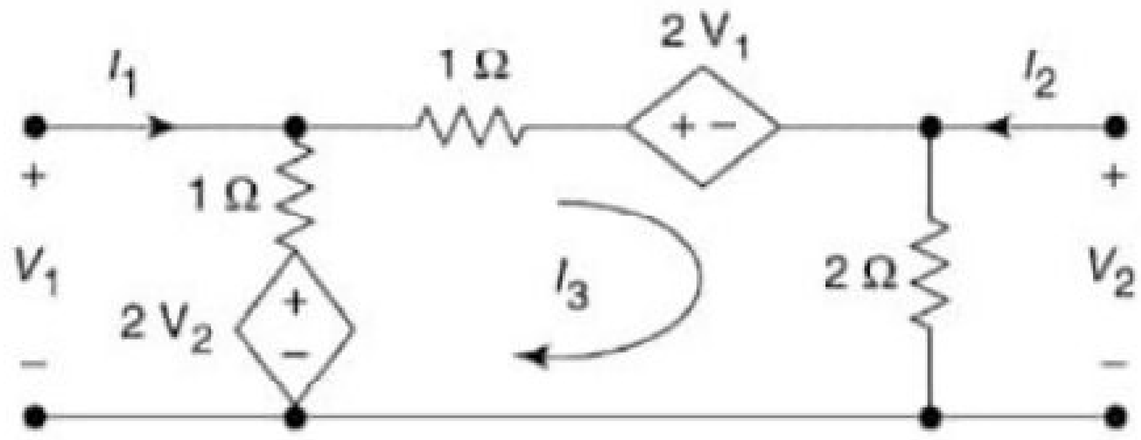
Therefore, the hybrid parameters are

$$h_{11} = \left[R_1 + \frac{(1-\alpha)R_2 R_3}{R_2 + R_3}\right]; \quad h_{12} = \frac{R_2}{R_2 + R_3}; \quad h_{21} = -\left(\frac{R_2 + \alpha R_3}{R_2 + R_3}\right); \quad h_{22} = \frac{1}{R_2 + R_3} \qquad \textit{Ans.}$$

7.44 Determine the y and z parameters for the network shown in the figure.



Solution We convert the dependent current source into equivalent dependent voltage source. The modified network is shown in the figure.



By KVL for three meshes, we get,

$$V_1 = 1 \times (I_1 - I_3) + 2V_2 \Rightarrow I_3 = I_1 + 2V_2 - V_1 \quad \text{(i)}$$

and $\quad 1 \times I_3 - 2V_1 + 2(I_2 + I_3) - 2V_2 + 1 \times (I_3 - I_1) = 0 \Rightarrow 2V_1 + 2V_2 = -I_1 + 2I_2 + 4I_3 \quad$ (ii)

and,
$$V_2 = 2 \times (I_2 + I_3) \quad \text{(iii)}$$

Substituting the value of I_3 from (i) into (ii) and (iii), we get,

$$2V_1 + 2V_2 = -I_1 + 2I_2 + 4(I_1 + 2V_2 - V_1) \Rightarrow 6V_1 - 6V_2 = 3I_1 + 2I_2 \quad \text{(iv)}$$

and,
$$V_2 = 2(I_2 + I_1 + 2V_2 - V_1) \Rightarrow 2V_1 - 3V_2 = 2I_1 + 2I_2 \quad \text{(v)}$$

By (iv) – (v), we get,

$$I_1 = 4V_1 - 3V_2 \quad \text{(vi)}$$

Also, from (v) and (vi), we get,

$$2V_1 - 3V_2 = 2(4V_1 - 3V_2) + 2I_2$$

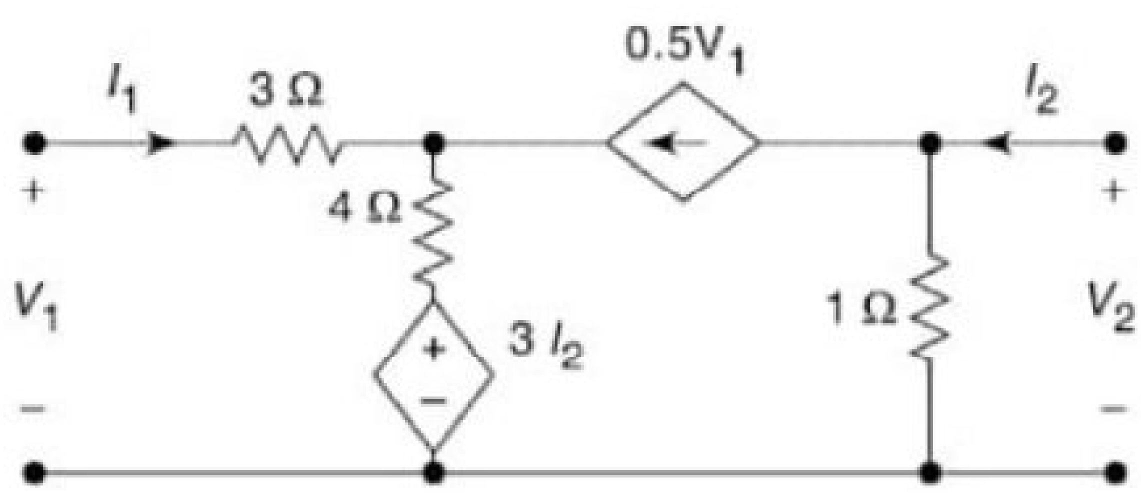
$$\Rightarrow \quad I_2 = -3V_1 + \frac{3}{2}V_2 \quad \text{(vii)}$$

From (vi) and (vii), we get,

$$y = \begin{bmatrix} 4 & -3 \\ -3 & \frac{3}{2} \end{bmatrix} (\text{mho}) \quad \textit{Ans.}$$

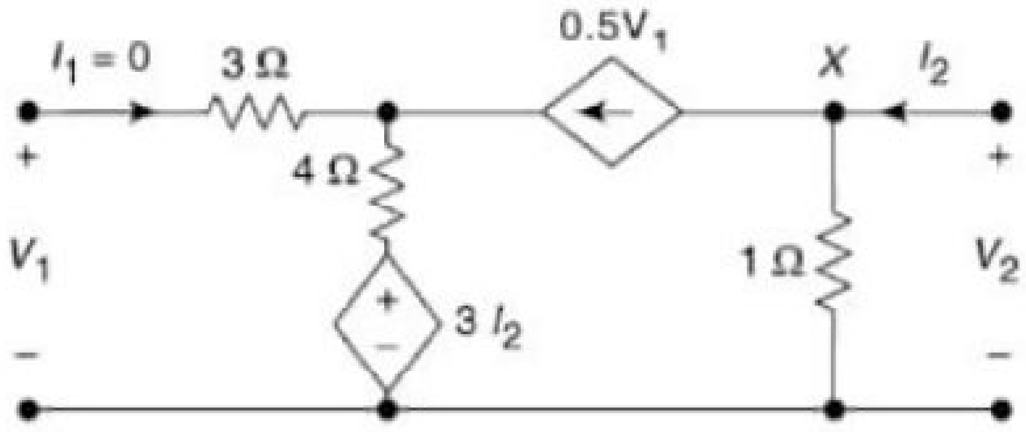
$$\therefore \quad z = [y]^{-1} = \begin{bmatrix} 4 & -3 \\ -3 & \frac{3}{2} \end{bmatrix}^{-1} = \begin{bmatrix} -\frac{1}{2} & -1 \\ -1 & -\frac{4}{3} \end{bmatrix} (\Omega) \quad \textit{Ans.}$$

7.45 Find the h-parameters for the two-port network shown in the figure.



Solution To find h parameters, we consider two cases:

When $I_1 = 0$ Here, no current will flow through the 3 Ω resistance.



By KVL at the left mesh, we get,

$$V_1 = 4 \times (0.5V_1) + 3I_2$$
$$= 2V_1 + 3I_2$$

$\Rightarrow$ $$V_1 = -3I_2$$

Also, by KCL at Node (X), we get,

$$I_2 = \frac{V_2}{1} + 0.5V_1 = V_2 + 0.5V_1$$
$$= V_2 + 0.5 \times (-3I_2)$$

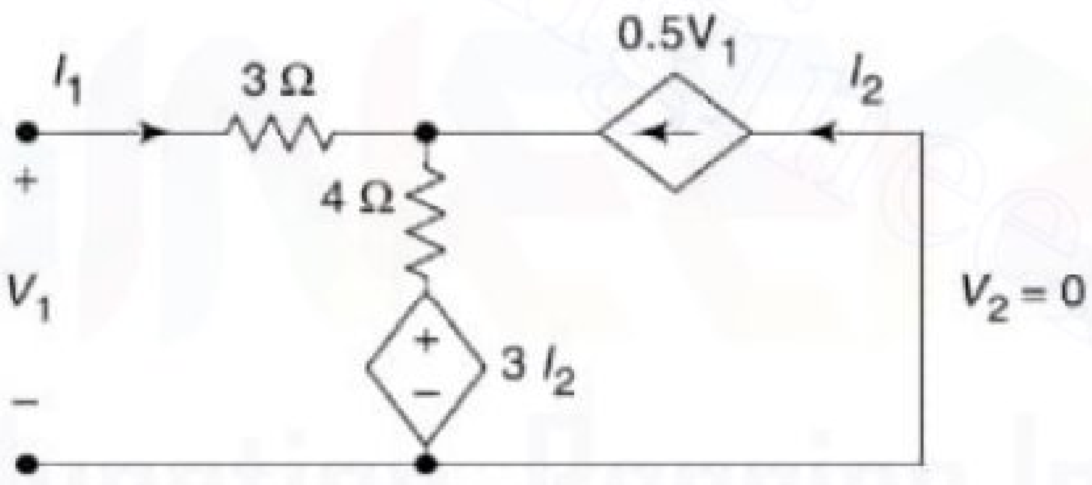
$\Rightarrow$ $$2.5I_2 = V_2$$

$\therefore$ $$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{2.5} = 0.4 \ \mho$$

$\therefore$ $$V_1 = -3I_2 = -3 \times \left(\frac{V_2}{2.5}\right) = -1.2V_2$$

$\therefore$ $$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = -1.2$$

When $V_2 = 0$ Here, the port 2 is short circuited. The 1Ω resistance becomes redundant. The modified circuit is shown in Fig (b).



(b)

$\therefore$ $$I_2 = 0.5V_1$$
$$= 0.5 \times [3I_1 + 4I_1 + 4I_2 + 3I_2]$$
$$= 3.5I_1 + 3.5I_2$$

$\Rightarrow$ $$2.5I_2 = -3.5I_1$$

$\therefore$ $$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -\frac{3.5}{2.5} = -1.4$$

Also,

$$V_1 = 3I_1 + 4I_1 + 4I_2 + 3I_2 = 7I_1 + 7I_2 = 7I_1 + 7 \times (-1.4I_1)$$
$$= -2.8I_1$$

$\therefore$ $$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = -2.8 \ \Omega$$

Therefore, the h parameters of the network are given as,

$$[h]=\begin{bmatrix}-2.8 & -1.2\\ -1.4 & 0.4\end{bmatrix} \quad \textit{Ans.}$$

MULTIPLE-CHOICE QUESTIONS

7.1 Which one of the following pairs is correctly matched?
(a) Symmetrical two-port network: $AD - BC = 1$
(b) Reciprocal two-port network: $z_{11} = z_{22}$.
(c) Inverse hybrid parameters: A, B, C, D
(d) Hybrid parameters: $(V_1, I_2) = f(I_1, V_2)$

7.2 What is the condition for reciprocity in terms of h-parameters?
(a) $h_{11} = h_{22}$ (b) $h_{12}h_{21} = h_{11}h_{22}$ (c) $h_{12} + h_{21} = 0$ (d) $h_{12} = h_{21}$

7.3 For a reciprocal network, the two-port $ABCD$ parameters are related as follows
(a) $AD - BC = 1$ (b) $AD - BC = 0$ (c) $AC - BD = 0$ (d) $AC - BD = 1$

7.4 For a symmetrical two port network
(a) $z_{11} = z_{22}$ (b) $z_{12} = z_{21}$ (c) $z_{11}z_{22} - z_{12}^2 = 0$ (d) $z_{11} = z_{22}$ and $z_{12} = z_{21}$

7.5 For a two port network to be reciprocal, it is necessary that
(a) $z_{11} = z_{22}$ and $y_{12} = y_{21}$ (b) $z_{11} = z_{22}$ and $AD - BC = 0$.
(c) $h_{21} = -h_{12}$ and $AD - BC = 0$ (d) $y_{12} = y_{21}$ and $h_{21} = -h_{12}$

7.6 A two port network is symmetrical if
(a) $z_{11}z_{22} - z_{12}z_{21} = 1$ (b) $AD - BC = 1$ (c) $h_{11}h_{22} - h_{12}h_{21} = 1$ (d) $y_{11}y_{22} - y_{12}y_{21} = 1$

7.7 A two port network is reciprocal if and only if
(a) $z_{11} = z_{22}$ (b) $BC - AD = -1$ (c) $y_{12} = -y_{21}$ (d) $h_{12} = h_{21}$

7.8 In terms of ABCD parameters, a two port network is symmetrical if and only if:
(a) $A = B$ (b) $B = C$ (c) $C = D$ (d) $D = A$

7.9 The condition for reciprocity of a two port network having different parameters are:
1. $h_{12} = -h_{21}$ 2. $g_{12} = -g_{21}$ 3. $A = D$
Choose the correct combination.
(a) 1 and 2 (b) 1 and 3 (c) 2 and 3 (d) 1, 2 and 3.

7.10 Two two-port networks with transmission parameters A_1, B_1, C_1, D_1 and A_2, B_2, C_2, D_2 respectively are cascaded. The transmission parameter matrix of the cascaded network will be

(a) $\begin{bmatrix}A_1 & B_1\\ C_1 & D_1\end{bmatrix}+\begin{bmatrix}A_2 & B_2\\ C_2 & D_2\end{bmatrix}$ (b) $\begin{bmatrix}A_1 & B_1\\ C_1 & D_1\end{bmatrix}\begin{bmatrix}A_2 & B_2\\ C_2 & D_2\end{bmatrix}$

(c) $\begin{bmatrix}A_1A_2 & B_1B_2\\ C_1C_2 & D_1D_2\end{bmatrix}$ (d) $\begin{bmatrix}(A_1A_2 + C_1C_2) & (A_1A_2 - B_1D_2)\\ (C_1A_2 - D_1C_2) & (C_1C_2 + D_1D_2)\end{bmatrix}$

7.11 Consider the following statements.
For a bilateral network,
1. $A = D$ 2. $z_{12} = z_{21}$ 3. $h_{12} = -h_{21}$
Of these statements.
(a) 1, 2 and 3 are correct (b) 1 and 2 are correct
(c) 1 and 3 are correct (d) 2 and 3 are correct.

7.12 In a two port network containing linear bilateral passive circuit elements, which one of the following conditions for z parameters would hold?

(a) $z_{11} = z_{22}$ (b) $z_{12}z_{21} = z_{11}z_{22}$ (c) $z_{11}z_{12} = z_{22}z_{21}$ (d) $z_{12} = z_{21}$

7.13 The relation $AD - BC = 1$, where A, B, C and D are the elements of a transmission matrix of a network, is valid for

(a) any type of network. (b) passive but not reciprocal network.
(c) passive and reciprocal network. (d) both active and passive network.

7.14 When a number of 2-port networks are connected in cascade, the individual:

(a) Z_{oc} matrices are added. (b) Y_{sc} matrices are added.
(c) chain matrices are multiplied. (d) H-matrices are multiplied.

7.15 The h parameters h_{11} and h_{22} are related to z and y parameters as

(a) $h_{11} = z_{11}$ and $h_{22} = \frac{1}{z_{22}}$ (b) $h_{11} = z_{11}$ and $h_{22} = y_{22}$
(c) $h_{11} = \frac{\Delta z}{z_{22}}$ and $h_{22} = \frac{1}{z_{22}}$ (d) $h_{11} = \frac{1}{y_{11}}$ and $h_{22} = y_{22}$

7.16 Two two-port networks α and β having A B C D parameters as

$A_\alpha = 4 = D_\alpha$ $A_B = 3 = D_B$ $B_\alpha = 5, C_\alpha = 3$ and $B_B = 4, C_B = 2$

are connected in cascade in the order of α, β. The equivalent A parameters of the combination is

(a) 17 (b) 22 (c) 24 (d) 31.

7.17 With the usual notation, a two-port resistive network satisfies the condition $A = D = \frac{3}{2}B = \frac{4}{3}C$

The z_{11} of the network is

(a) $\frac{5}{3}$ (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

7.18 The reciprocal of a network function is

(a) an immittance function, if the original function is an immittance function.
(b) a transfer function, if the original function is a transfer function.
(c) never an immittance function.
(d) never a transfer function.

7.19 A two-port network is defined by the relations $I_1 = 2V_1 + V_2$, $I_2 = 2V_1 + 3V_2$. Then z_{12} is

(a) $-2\ \Omega$ (b) $-1\ \Omega$ (c) $-\frac{1}{2}\ \Omega$ (d) $-\frac{1}{4}\ \Omega$

7.20 Consider the following statements

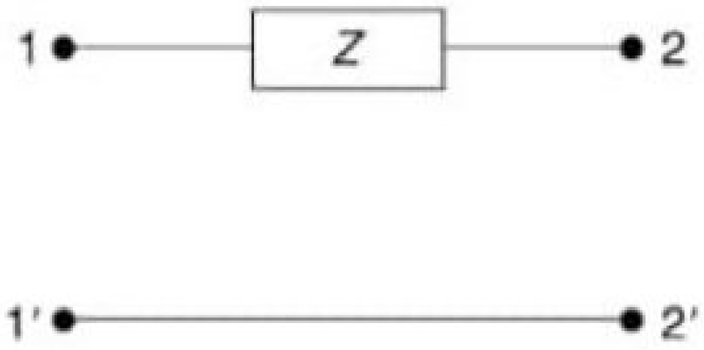
1. Transfer impedance is the reciprocal of transfer admittance.
2. One can derive transfer impedance of a network if its driving-point impedance and admittance are known.
3. Driving-point impedance is the ratio of the Laplace transform of voltage and current functions at the input.

Of these statements:

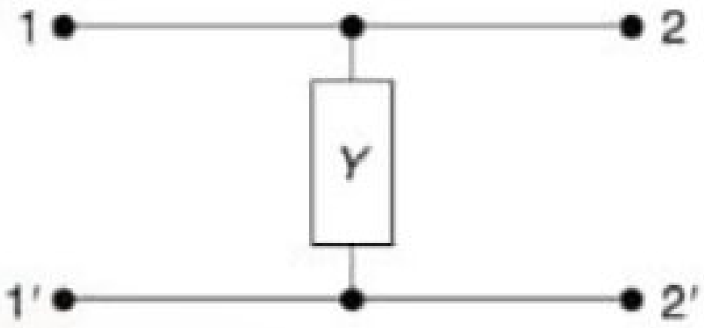
(a) 1, 2 and 3 are correct (b) 1 and 2 are correct
(c) 2 and 3 are correct (d) 3 alone is correct.

7.21 Consider the following statements

1. The two-port network shown below does NOT have an impedance matrix representation.



2. The two-port network shown below does NOT have an admittance matrix representation.



3. A two-port network is said to be reciprocal if it satisfies $z_{12} = z_{21}$ or an equivalent relationship.

Of these statements:

(a) 1 and 2 are correct (b) 1 and 3 are correct
(c) 1 and 3 are correct (d) None is correct.

7.22 If two two-port networks are connected in series, and if the port current requirement is satisfied, which of the following is true?

(a) The z-parameter matrices add (b) The y-parameter matrices add.
(c) The $ABCD$-parameter matrices add. (d) None of these.

7.23 If two two-port networks are connected in parallel, and if the port current requirement is satisfied, which of the following is true?

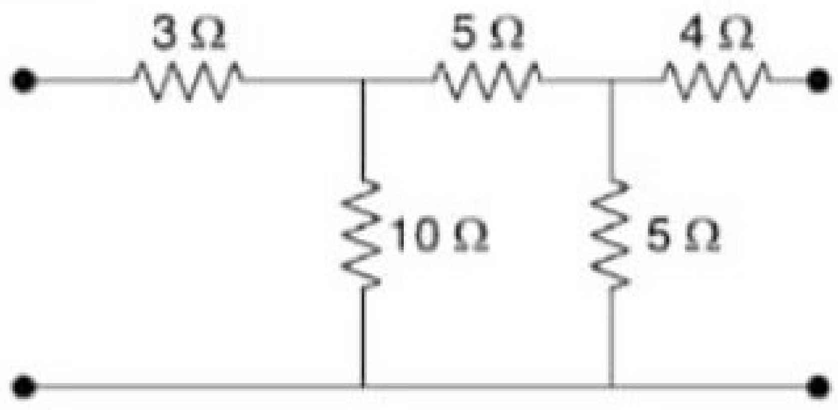
(a) The z-parameter matrices add (b) The y-parameter matrices add.
(c) The $ABCD$-parameter matrices add (d) None of these.

7.24 If two two-port networks are connected in cascade, and if the port current requirement is satisfied, which of the following is true?

(a) The z-parameter matrices add (b) The y-parameter matrices add.
(c) The $ABCD$-parameter matrices add (d) None of these.

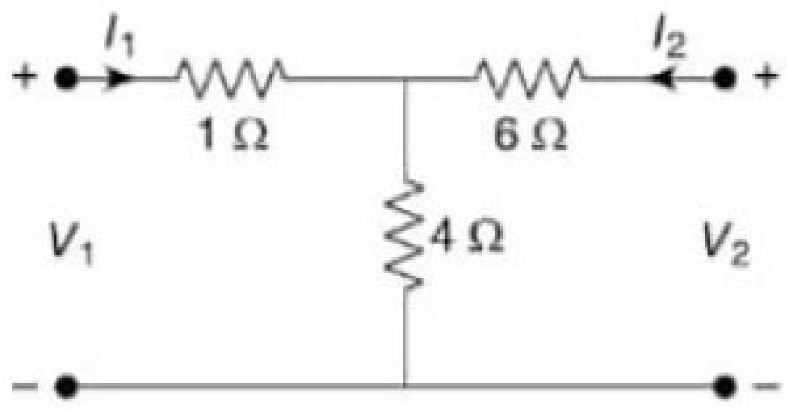
7.25 The z_{11} and z_{22} parameters of the given network are

(a) 8 Ω, 7.75 Ω
(b) 13 Ω, 9 Ω
(c) 12 Ω, 8.5 Ω
(d) None of the above.



7.26 For the network shown, the parameters h_{11} and h_{21} are

(a) 5 Ω and −2/3 Ω (b) 3.4 Ω and −2/5 Ω
(c) 3.4 Ω and −3/5 Ω (d) None of the above.



7.27 The maximum value of the transmission parameter A for a passive, reciprocal, linear two-port network is

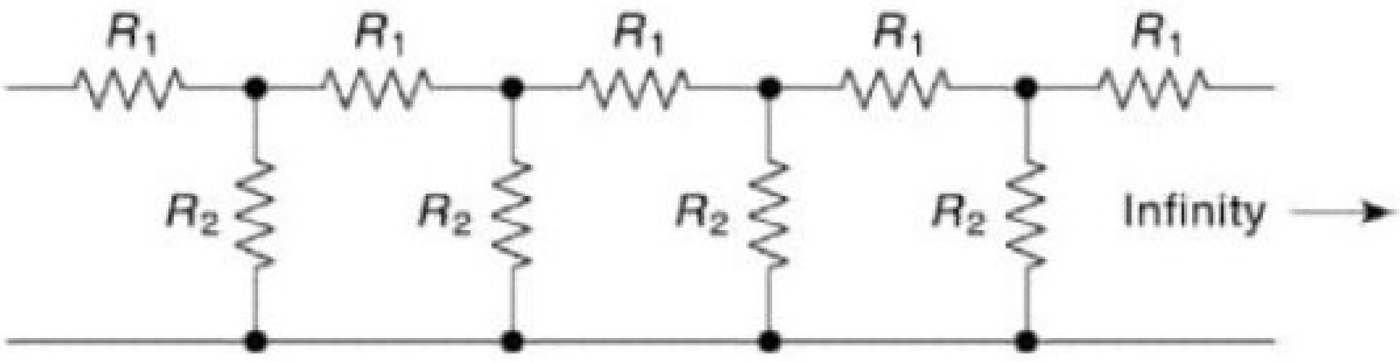
(a) 1 (b) 2
(c) 3 (d) none of the above.

7.28. The unique feature of *ABCD* parameters as compared to *x*, *y* and *h* parameters is

(a) none (b) short-circuit functions

(c) open-circuit functions (d) reverse transverse functions

7.29. The driving point impedance of the infinite ladder network shown in the given figure is



(given $R_1 = 2\ \Omega$ and $R_2 = 1.5\ \Omega$)

(a) $3\ \Omega$ (b) $3.5\ \Omega$ (c) $\frac{3}{3.5}\ \Omega$ (d) $\ln\left(1+\frac{3}{3.5}\right)\Omega$

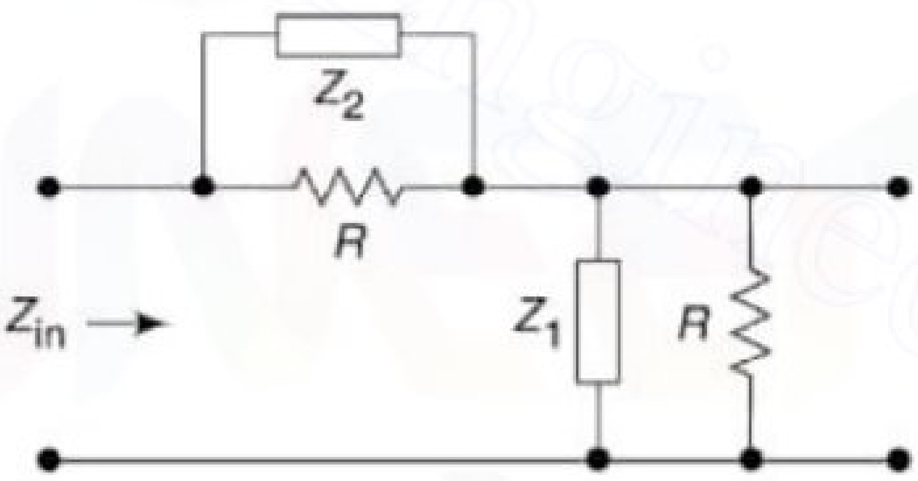
7.30 A 2-port network is described by the relations:

$$V_1 = 2V_2 + 0.5I_2$$
$$I_1 = 2V_2 + I_2$$

What is the value of the h_{22} parameter of the network?

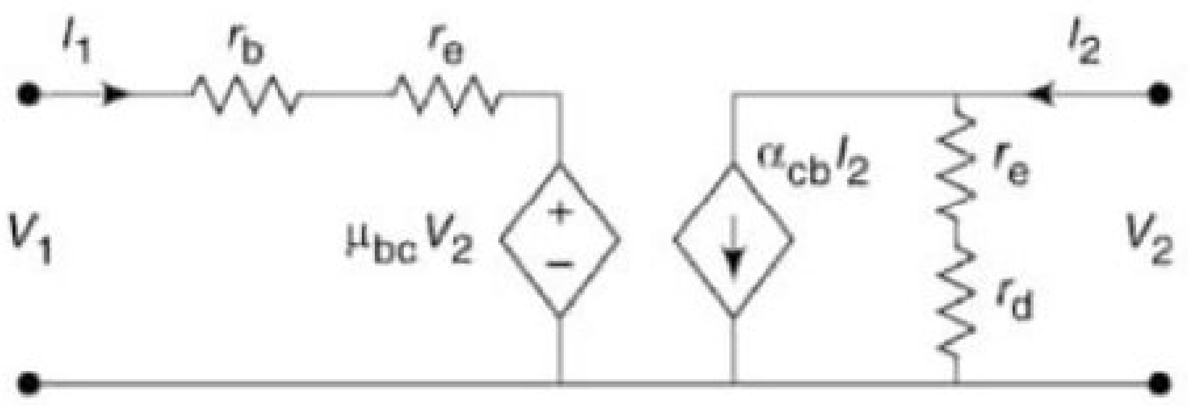
(a) 1 mho (b) 2Ω (c) -2 mho (d) 4Ω

7.31 What are the suitable values for Z_1 and Z_2, to make the input impedance, Z_{in}, of the network equal to *R*?



(a) *R* and *R* (b) 2*R* and *R* (c) 3*R* and 2*R* (d) 4*R* and 4*R*

7.32



Which one of the following gives the *h*-parameter matrix for the network shown in the figure?

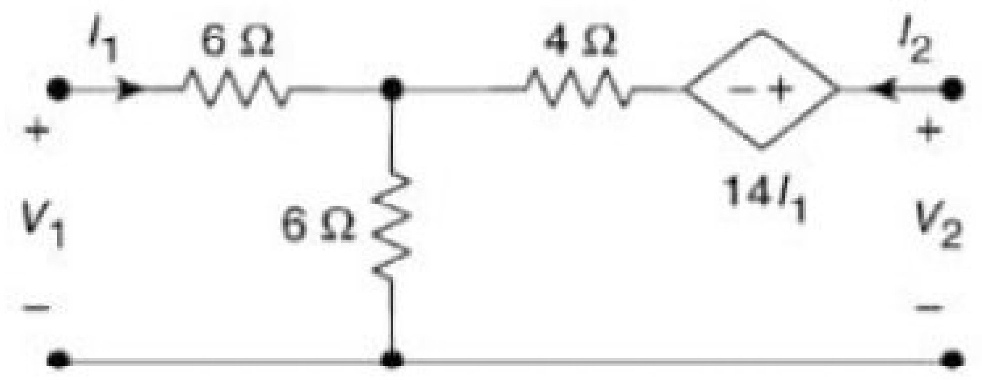
(a) $\begin{bmatrix} \frac{1}{r_e + r_d} & \mu_{bc} \\ \alpha_{cb} & r_b + r_e \end{bmatrix}$ (b) $\begin{bmatrix} r_b + r_e & \alpha_{cb} \\ \mu_{bc} & \frac{1}{r_e + r_d} \end{bmatrix}$

(c) $\begin{bmatrix} r_b + r_e & \mu_{bc} \\ \alpha_{cb} & \frac{1}{r_e + r_d} \end{bmatrix}$ (d) $\begin{bmatrix} \mu_{bc} & \alpha_{cb} \\ r_b + r_e & \frac{1}{r_e + r_d} \end{bmatrix}$

7.33 In a two-port network, the output short-circuit current was measured while the source voltage at the input was 1 V; the value of the output current would provide the parameter

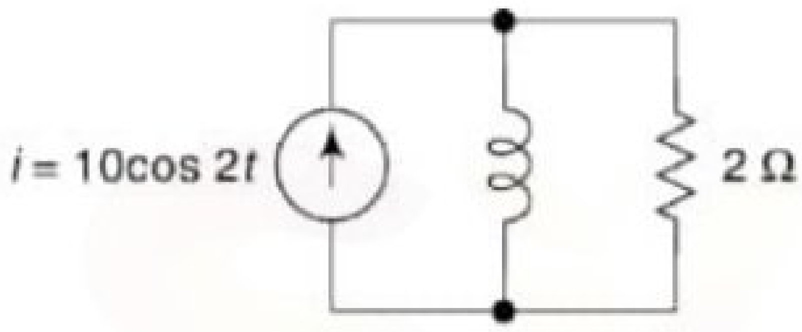
(a) B (b) y_{12} (c) h_{21} (d) y_{21}

7.34 The y-parameter 'y_{21}' of the network shown in the figure



(a) is 2 mho (b) is 6 mho (c) is 3 mho (d) does not exist

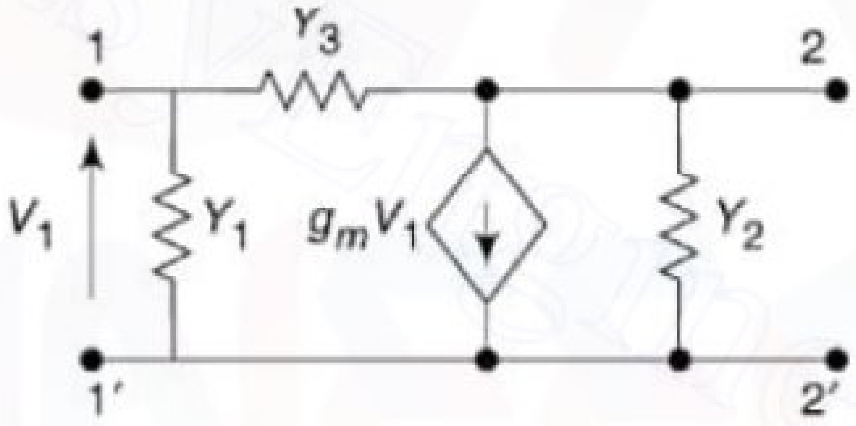
7.35 The phasor current through the inductance in the circuit shown is



(a) $\left(\frac{10}{\sqrt{2}}\right)\angle -45^\circ$ (b) $\left(\frac{10}{\sqrt{2}}\right)\angle 45^\circ$

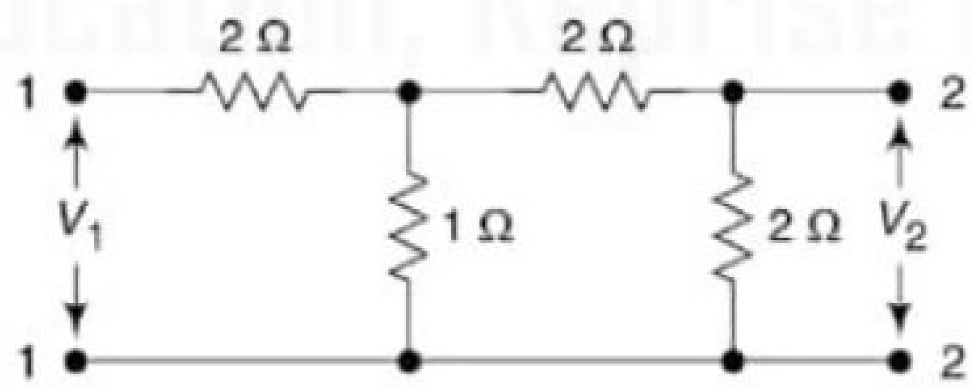
(c) $5\angle 45^\circ$ (d) $5\angle -45^\circ$

7.36 For the two-port network, the parameter y_{21} will be



(a) $Y_2 + Y_3$ (b) $g_m - Y_3$ (c) $Y_3 - g_m$ (d) $g_m + Y_2 + Y_3$

7.37 For the given two-port network, z_{21} will be

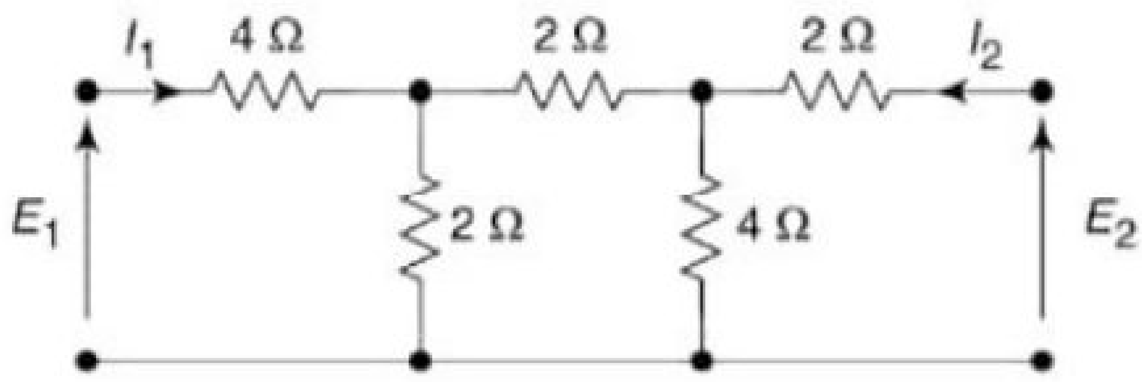


(a) 2/5 Ω (b) 3/5 Ω (c) 1/5 Ω (d) 4/5 Ω

7.38 The h-parameters for a two-port network are defined by $\begin{bmatrix} E_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ E_2 \end{bmatrix}$. For the two-port network shown in the figure, the value of h_{12} is given by

(a) 0.125 (b) 0.167

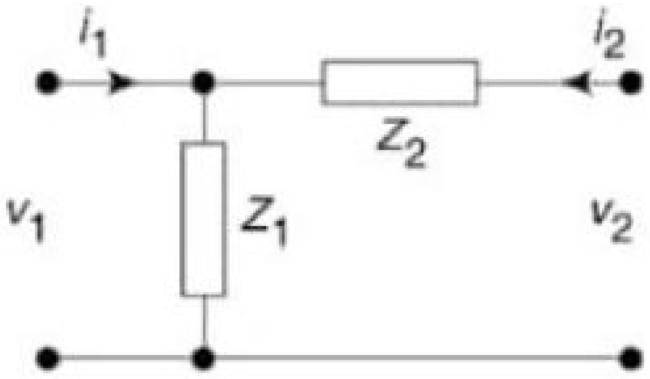
(c) 0.625 (d) 0.25



7.39 The z matrix of a two-port network as given by $\begin{bmatrix} 0.9 & 0.2 \\ 0.2 & 0.6 \end{bmatrix}$. The element y_{22} of the corresponding y matrix of the same network is given by

(a) 1.2 (b) 0.4 (c) −0.4 (d) 1.8

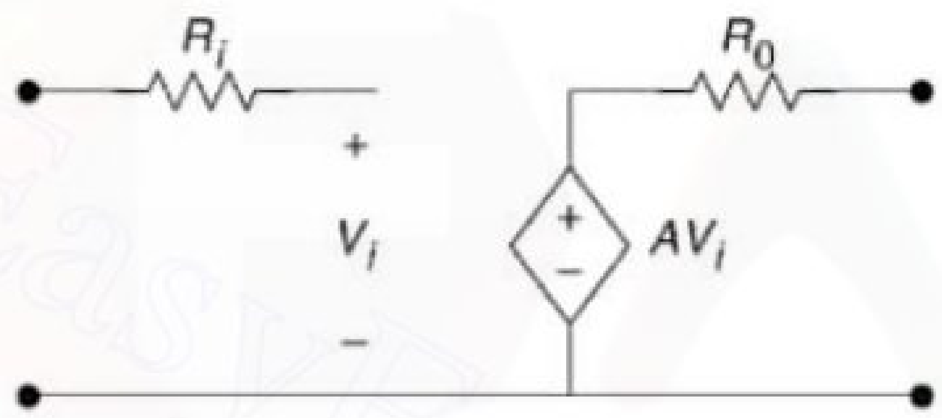
7.40 For the two-port network shown in the figure, the z-matrix is given by



(a) $\begin{bmatrix} Z_1 & Z_1+Z_2 \\ Z_1+Z_2 & Z_2 \end{bmatrix}$ (b) $\begin{bmatrix} Z_1 & Z_1 \\ Z_1+Z_2 & Z_2 \end{bmatrix}$

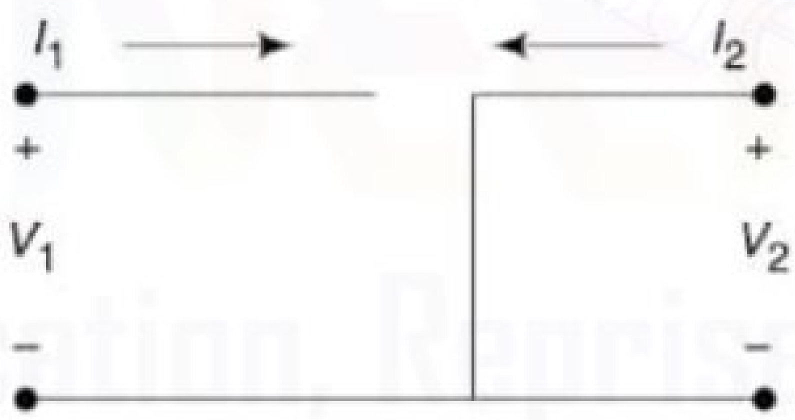
(c) $\begin{bmatrix} Z_1 & Z_2 \\ Z_2 & Z_1+Z_2 \end{bmatrix}$ (d) $\begin{bmatrix} Z_1 & Z_1 \\ Z_1 & Z_1+Z_2 \end{bmatrix}$

7.41 The parameters of the circuit shown in the figure are $R_i = 1$ M Ω, $R_0 = 10$ Ω, $A = 10^6$ V/V. If $V_i = 1$ μV, then output voltage, input impedance and output impedance respectively are



(a) 1 V, ∞, 10 Ω (b) 1 V, 0, 10 Ω (c) 1 V, 0, ∞ (d) 10 V, ∞, 10 Ω

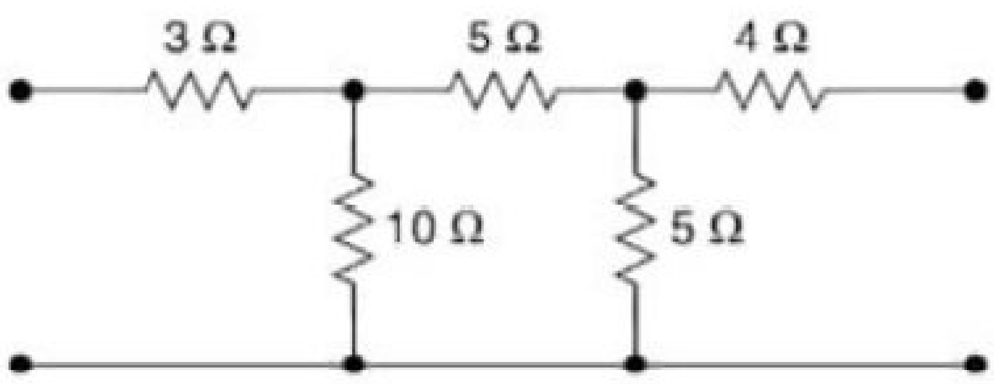
7.42 The parameter type and the matrix representation of the relevant two port parameters that describe the circuit shown are



(a) z parameters, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (b) h parameters, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

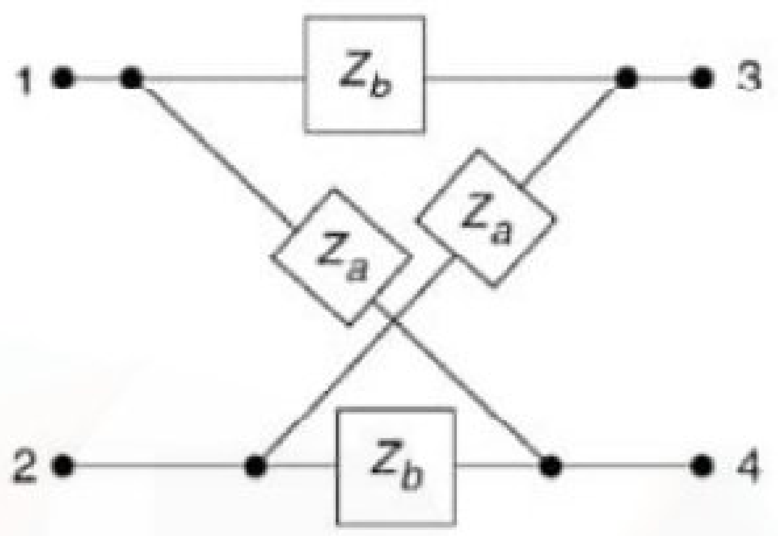
(c) h parameters, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) z parameters, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

7.43 The impedance parameters z_{11} and z_{12} of the two-port network in the figure are



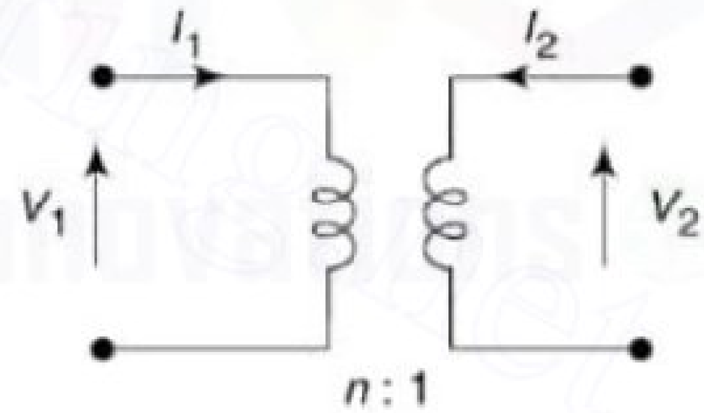
(a) $z_{11} = 2.75\ \Omega$ and $z_{12} = 0.25\ \Omega$
(b) $z_{11} = 3\ \Omega$ and $z_{12} = 0.5\ \Omega$
(c) $z_{11} = 3\ \Omega$ and $z_{12} = 0.25\ \Omega$
(d) $z_{11} = 2.25\ \Omega$ and $z_{12} = 0.5\ \Omega$

7.44 For the lattice circuit shown in the figure, $Z_a = j2\ \Omega$ and $Z_b = 2\ \Omega$. The values of the open circuit impedance parameters $z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$ are



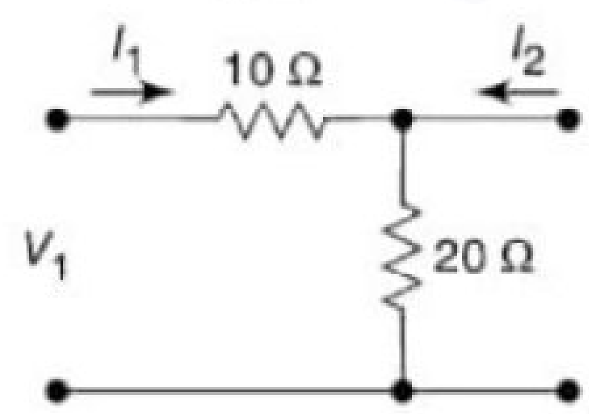
(a) $\begin{bmatrix} 1-j & 1+j \\ 1+j & 1+j \end{bmatrix}$
(b) $\begin{bmatrix} 1-j & 1+j \\ -1+j & 1-j \end{bmatrix}$
(c) $\begin{bmatrix} 1+j & 1+j \\ 1-j & 1-j \end{bmatrix}$
(d) $\begin{bmatrix} 1-j & -1+j \\ -1-j & 1-j \end{bmatrix}$

7.45 The *ABCD* parameters of an ideal $n:1$ transformer shown in the figure are $\begin{bmatrix} n & 0 \\ 0 & X \end{bmatrix}$. The value of X will be



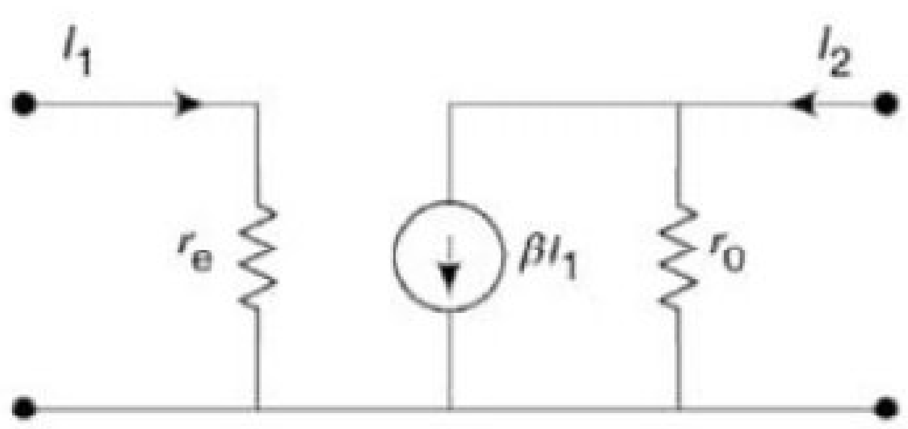
(a) n
(b) $\frac{1}{n}$
(c) n^2
(d) $\frac{1}{n^2}$

7.46 The *h*-parameters of the circuit shown in the figure are



(a) $\begin{bmatrix} 0.1 & 0.1 \\ -0.1 & 0.3 \end{bmatrix}$
(b) $\begin{bmatrix} 10 & -1 \\ 1 & 0.05 \end{bmatrix}$
(c) $\begin{bmatrix} 30 & 20 \\ 20 & 20 \end{bmatrix}$
(d) $\begin{bmatrix} 10 & 1 \\ -1 & 0.05 \end{bmatrix}$

7.47 In the two-port network shown in the figure below, z_{12} and z_{21} are, respectively,



(a) r_e and βr_0 (b) 0 and $-\beta r_0$ (c) 0 and βr_0 (d) r_e and $-\beta r_0$

7.48 A two-port network is represented by $ABCD$ parameters given by

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

If port-2 is terminated by R_L, then the input impedance seen at port-1 given by

(a) $\dfrac{A + BR_L}{C + DR_L}$ (b) $\dfrac{AR_L + C}{BR_L + D}$

(c) $\dfrac{DR_L + A}{BR_L + C}$ (d) $\dfrac{B + AR_L}{D + CR_L}$

EXERCISES

7.1 Current I_1 and I_2 entering at ports 1 and 2 respectively of a two-port network are given by the following equations:

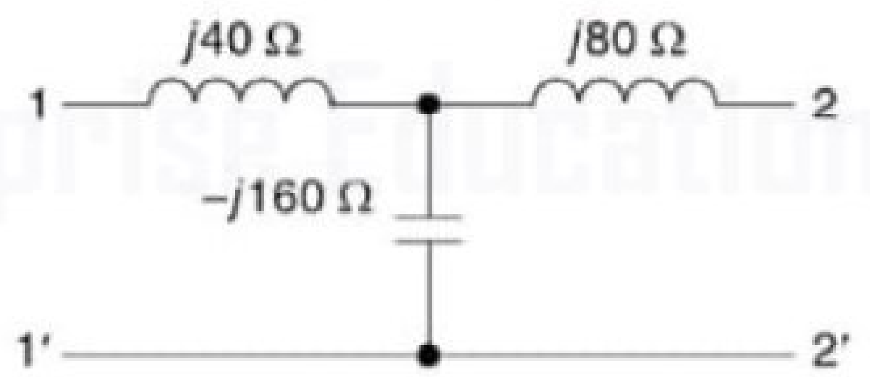
$$I_1 = 0.5V_1 - 0.2V_2$$
$$I_2 = -0.2V_1 + V_2$$

where V_1 and V_2 are the voltages at ports 1 and 2 respectively. Find the y, z and $ABCD$ parameters for the network. Also find its equivalent π-network.

[$y_{11} = 0.5$ ℧; $y_{12} = -0.2$ ℧; $y_{21} = -0.2$ ℧; $y_{22} = 1$ ℧;
$z_{11} = 2.174\ \Omega$; $z_{12} = z_{21} = -0.435\ \Omega$; $z_{22} = 1.086\ \Omega$;
$A = 5$, $B = 5\ \Omega$, $C = 2.3$ ℧, $D = 2.5$; $Y_1 = 0.3$ ℧; $Y_2 = 0.2$ ℧; $Y_3 = 0.8$ ℧]

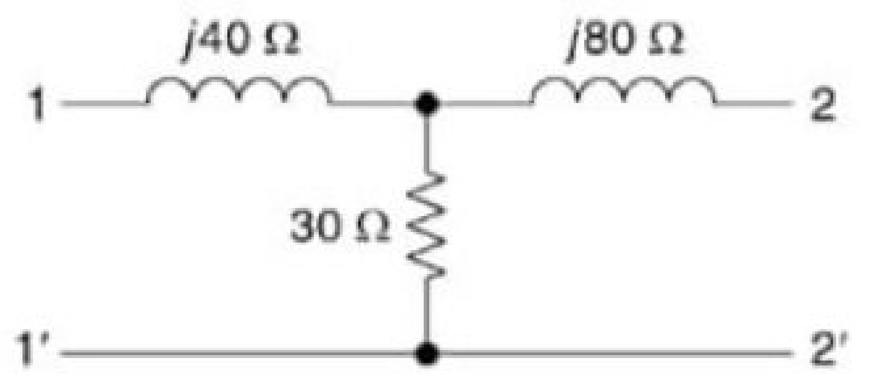
7.2 Determine the z-and y-parameters of the networks shown in figure.

(a)



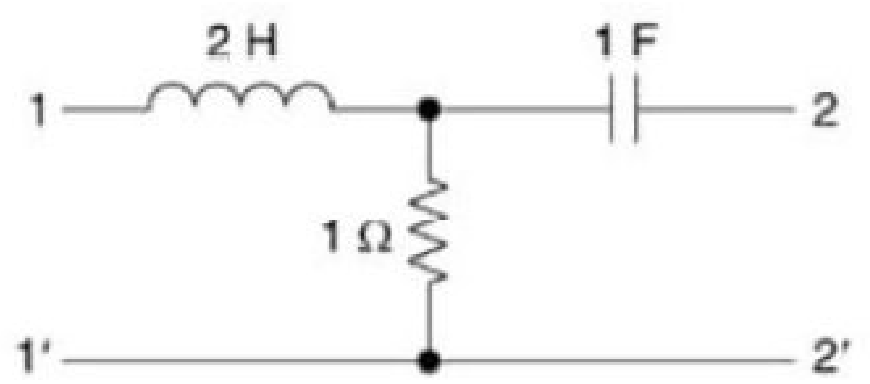
$$\left\{ z = \begin{bmatrix} -j120 & -j160 \\ -j160 & -j80 \end{bmatrix} (\Omega);\ y = \begin{bmatrix} -j/120 & j110 \\ j/110 & -j\dfrac{3}{4} \end{bmatrix} (\Omega^{-1}) \right\}$$

(b)



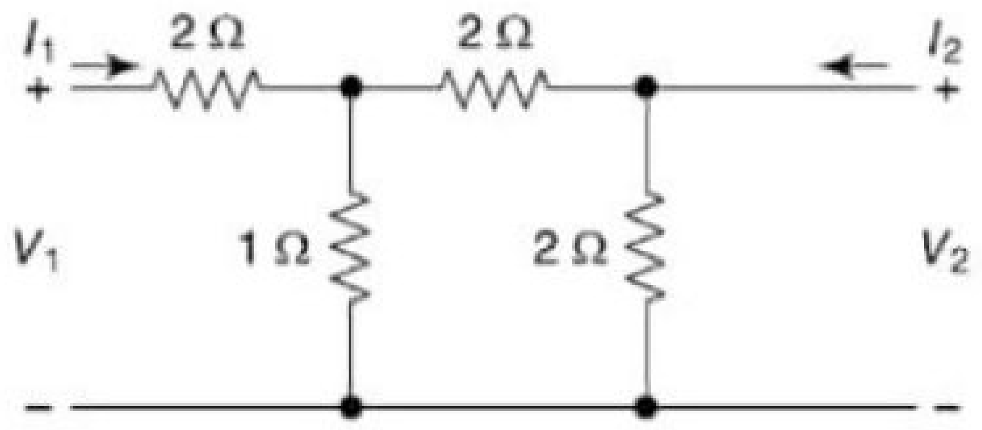
$$\left\{ z = \begin{bmatrix} (30 + j40) & j40 \\ j40 & (30 + j80) \end{bmatrix} (\Omega) \right\}$$

(c)



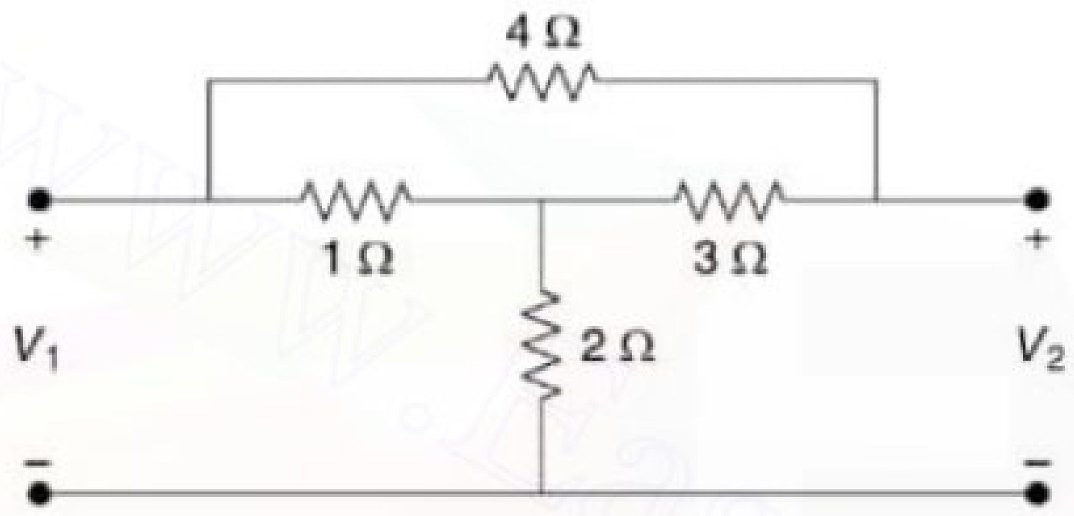
$$\left\{ z = \begin{bmatrix} (1 + 25) & 1 \\ 1 & \left(1 + \dfrac{1}{5}\right) \end{bmatrix} (\Omega) \right\}$$

7.3 Obtain the z-parameters for the circuit shown in figure and hence draw the z-parameter equivalent circuit.



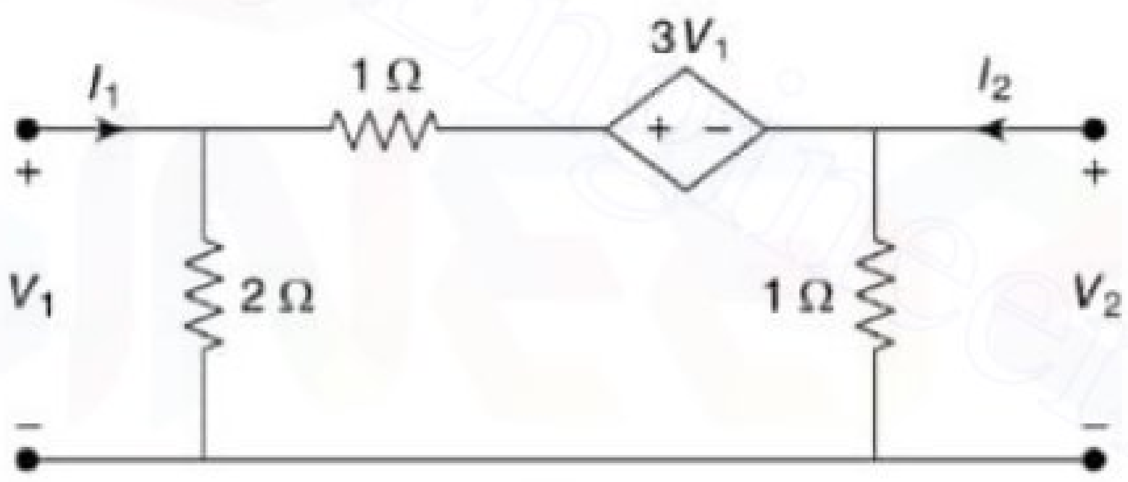
$$\left\{ z = \begin{bmatrix} \frac{14}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{6}{5} \end{bmatrix} (\Omega) \right\}$$

7.4 Find the open-circuit and short-circuit impedances of the network shown in figure.



$$\left[y = \begin{bmatrix} \frac{31}{44} & -\frac{19}{44} \\ -\frac{19}{44} & \frac{23}{44} \end{bmatrix}; z\text{-parameters do not exist} \right]$$

7.5 Find the z-parameters for the 2-port networks shown in figure containing a controlled source.



$$\left\{ z = \begin{bmatrix} -2 & -1 \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix} (\Omega) \right\}$$

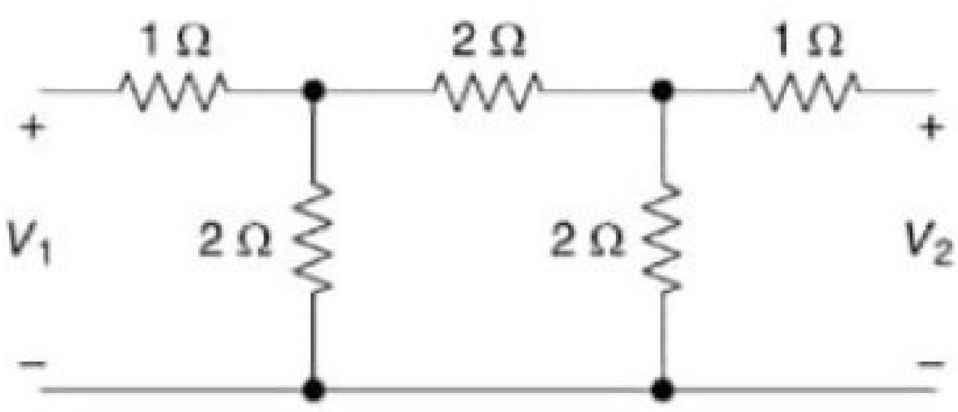
7.6 A 2-port network made up of passive linear resistors is fed at port 1 by an ideal voltage source of V volt. It is loaded at port 2 by a resistor R.

(i) With $V = 10$ volt and $R = 6\ \Omega$ currents at ports 1 and 2 were 1.44 A and 0.2 A respectively.

(ii) With $V = 15$ volt and $R = 8\ \Omega$ current at port 2 was 0.25 A.

Determine the π-equivalent circuit of the 2-port network. $\{Y_A = 0.2;\ Y_B = 0.3;\ Y_C = 0.5$ (mho)$\}$

7.7 Calculate the T-parameters for the block A and B separately and then using these results calculate the T-parameters of the whole circuit shown in figure. Prove any formula used.

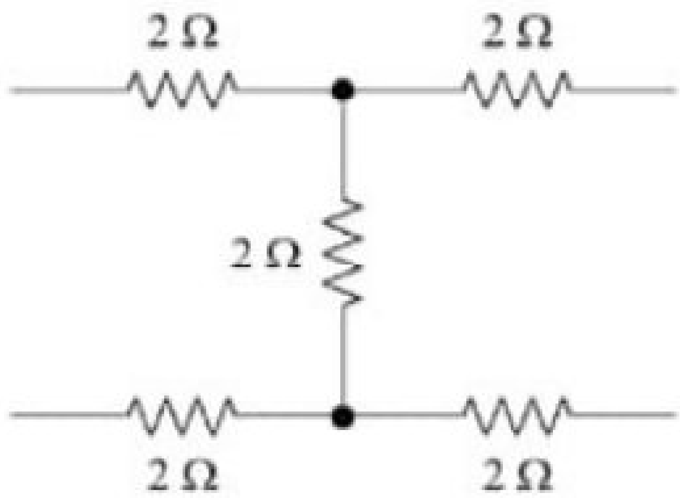


$$\left\{ A_a = A_b = D_a = D_b = \frac{3}{2} \right.$$

$$C_a = C_b = \frac{1}{2};\ B_a = B_b = \frac{5}{2};$$

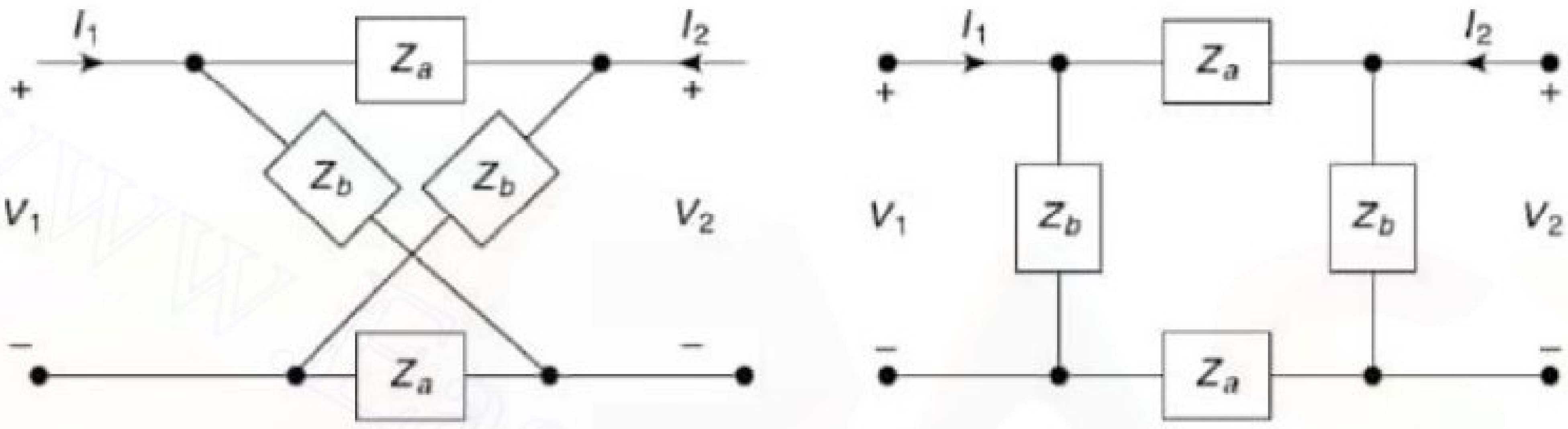
$$\left. T = \begin{bmatrix} 7/2 & 15/2 \\ 3/2 & 7/2 \end{bmatrix} \right\}$$

7.8 Find out the z-parameters of the two-port network shown in the figure.



$$\left\{ z = \begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix} (\Omega) \right\}$$

7.9 Find the z-parameters for the lattice network shown in the figure.



$$\left\{ z_{11} = z_{22} = \left(\frac{Z_b + Z_a}{2} \right); z_{12} = z_{21} = \left(\frac{Z_b - Z_a}{2} \right); \right.$$

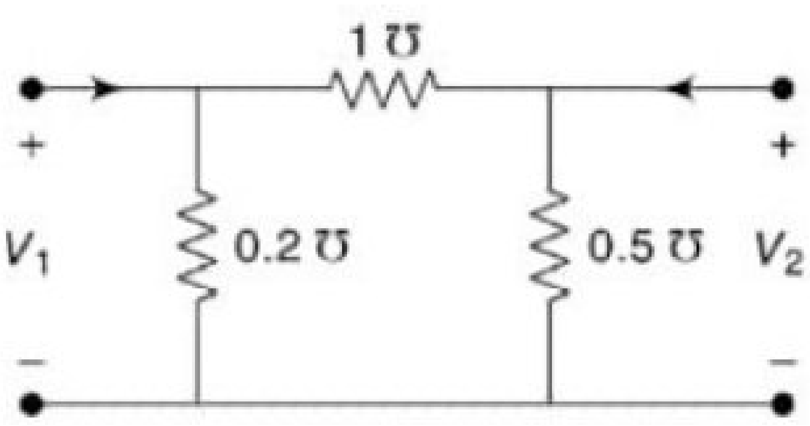
$$\left. z = \frac{1}{2Z_a + Z_b} \begin{bmatrix} 2(Z_a + Z_b) & Z_b^2 \\ Z_b^2 & 2Z_a Z_b \end{bmatrix} \right\}$$

7.10 Current I_1 and I_2 entering at port-1 and port-2 respectively of a two port network are given by the following equations: I_1=0.5V_1-0.2V_2, I_2=-0.2V_1+V_2, where V_1 and V_2 are the voltages at port-1 and port-2 respectively. Find the y, z and $ABCD$ parameters for the network. Also find the equivalent π-network.

$$\left\{ y = \begin{bmatrix} 0.5 & -0.2 \\ -0.2 & 1 \end{bmatrix} (\Omega^{-1}); Z = \begin{bmatrix} 2.174 & 0.435 \\ 0.435 & 1.087 \end{bmatrix} (\Omega), \right.$$

$$\left. T = \begin{bmatrix} 5 & 5\,\Omega \\ 2.3\,℧ & 2.5 \end{bmatrix}; Y_a = 0.3\,℧, Y_b = 0.8\,℧, Y_c = 0.2\,℧ \right\}$$

7.11 Two identical sections of the circuit shown in the figure are connected in series. Obtain the z-parameters of the combination and verify by direct calculation. $[z_{11} = z_{22} = 6\,\Omega; z_{12} = z_{21} = 4\,\Omega]$



7.12 Test results for a two-port network are

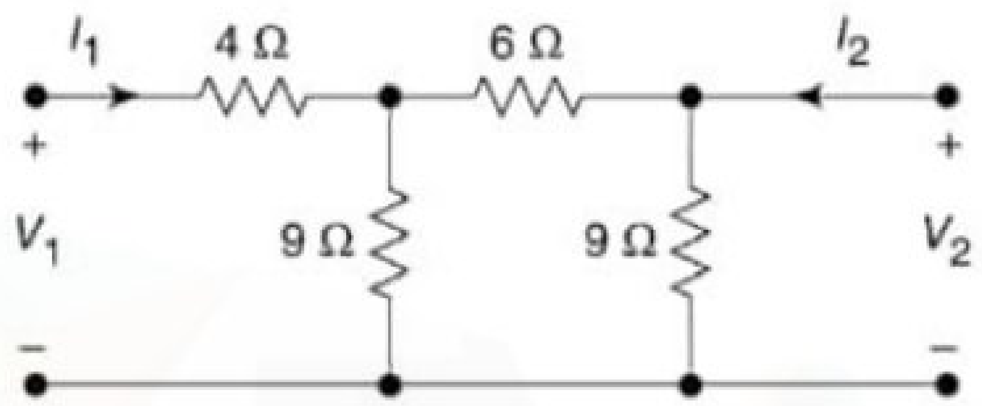
(a) port 2 open-circuited, $I_1 = 0.01\angle 0^\circ(\text{A}), V_1 = 1.4\angle 45^\circ(V), V_2 = 2.3\angle -26.4^\circ(\text{V})$

(b) port 1 open-circuited, $I_2 = 0.01\angle 0^\circ(\text{A}), V_1 = 1\angle -90^\circ(V), V_2 = 1.5\angle -53.1^\circ(\text{V})$

The source frequency in both the tests was 1000 Hz. Find z-parameters.

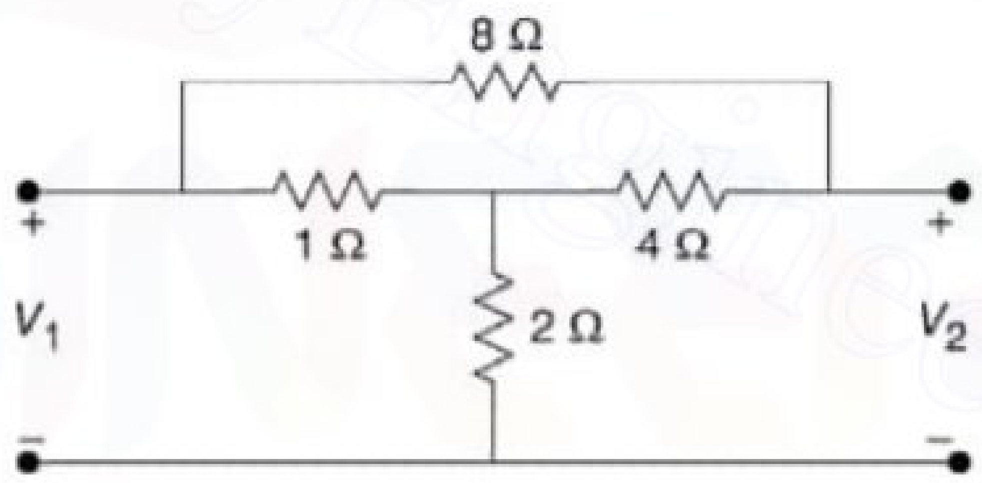
$$\left[\begin{bmatrix} 140\angle 45^\circ & 100\angle -90^\circ \\ 230\angle -26.4^\circ & 150\angle -53.1^\circ \end{bmatrix}(\Omega)\right]$$

7.13 Find the z-parameters for the network shown in the figure.



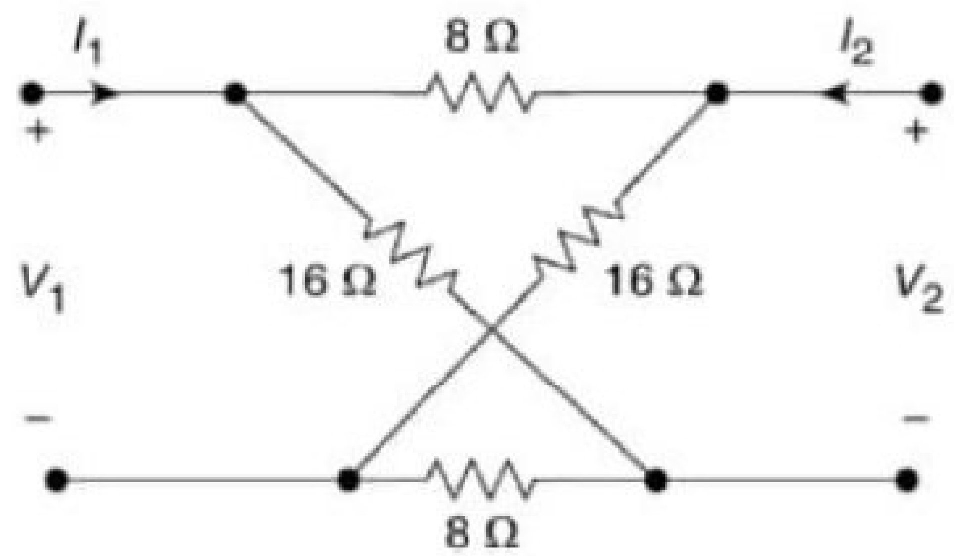
$$\left[\begin{bmatrix} 10 & 3 \\ 3 & 6 \end{bmatrix}(\Omega)\right]$$

7.14 For the network shown in the figure, find the y-parameters and also the equivalent T-network.



$$\left[\begin{bmatrix} 62/112 & -30/112 \\ -30/112 & 38/112 \end{bmatrix}, Z_a = 8/13\,\Omega, Z_b = 32/13\,\Omega, Z_c = 30/13\,\Omega\right]$$

7.15 Find the h-parameters for the network shown in the figure.



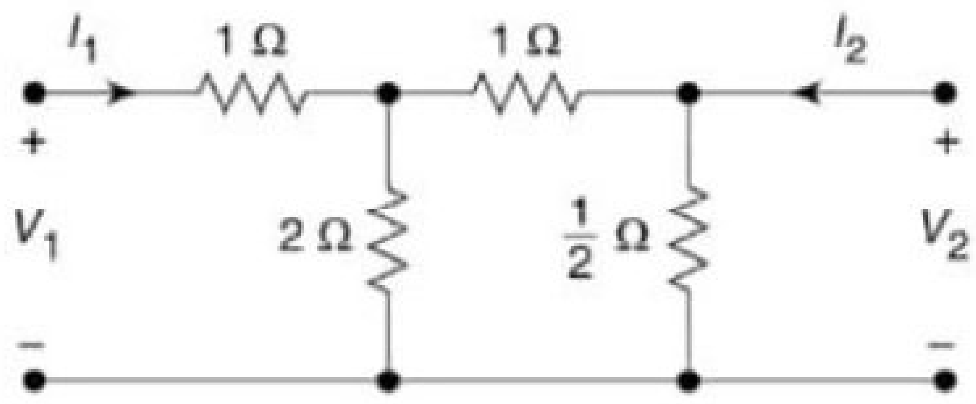
$$\left[h_{11} = \frac{32}{3}\,\Omega;\quad h_{12} = \frac{1}{3};\quad h_{21} = -\frac{1}{3};\quad h_{22} = \frac{1}{12}\,\text{mho}\right]$$

7.16 The h-parameters of a two-port network are

$$h_{11} = 35\Omega; \quad h_{12} = 2.6 \times 10^{-4}; \quad h_{21} = -0.98; \quad h_{22} = 0.3 \times 10^{-6}\,\text{mho}$$

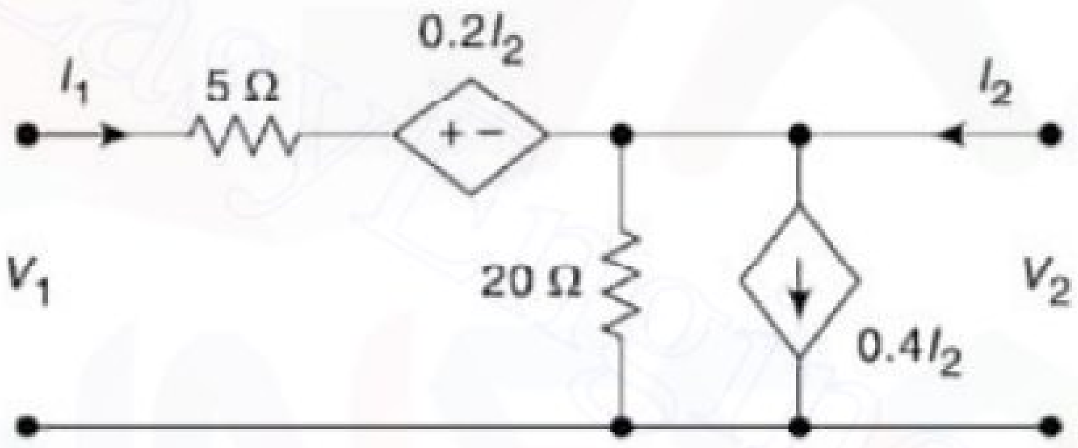
The input terminals are connected to 0.001V sinusoidal source and a 10^4 ohm resistance is connected across the output port. Find the output voltage. [0.26 V]

7.17 Find the y and z-parameters for the network shown in the figure.



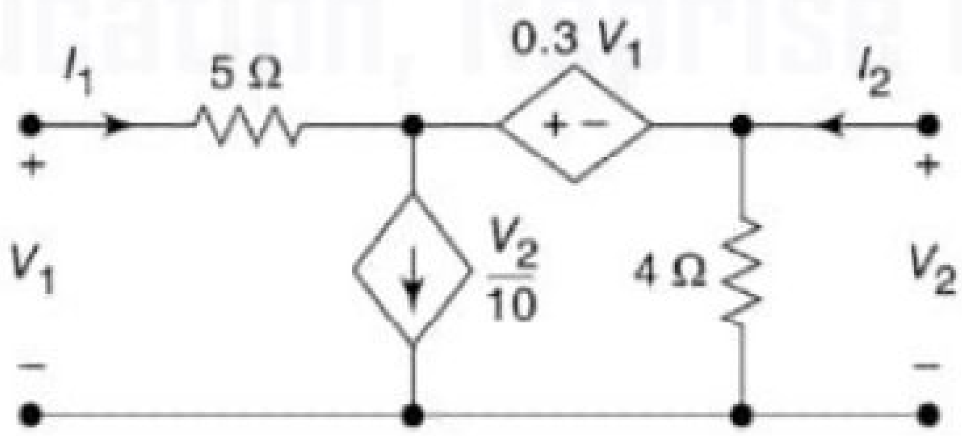
$$\left[\begin{bmatrix} 13/7 & 2/7 \\ 2/7 & 3/7 \end{bmatrix}(\Omega); \begin{bmatrix} -3/5 & -2/5 \\ -2/5 & 13/5 \end{bmatrix}(\text{mho})\right]$$

7.18 Find the y-parameters for the network shown in the figure.



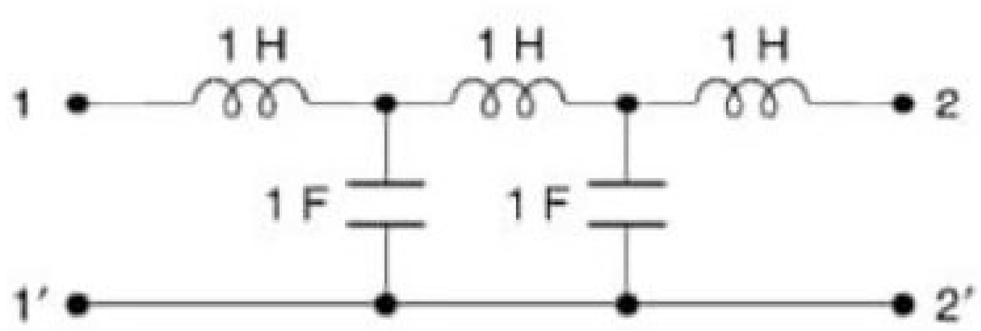
$$\left[\begin{bmatrix} 0.2 & -0.24 \\ -0.333 & 0.4833 \end{bmatrix}\mho\right]$$

7.19 Find the transmission parameters of the network shown in the figure.



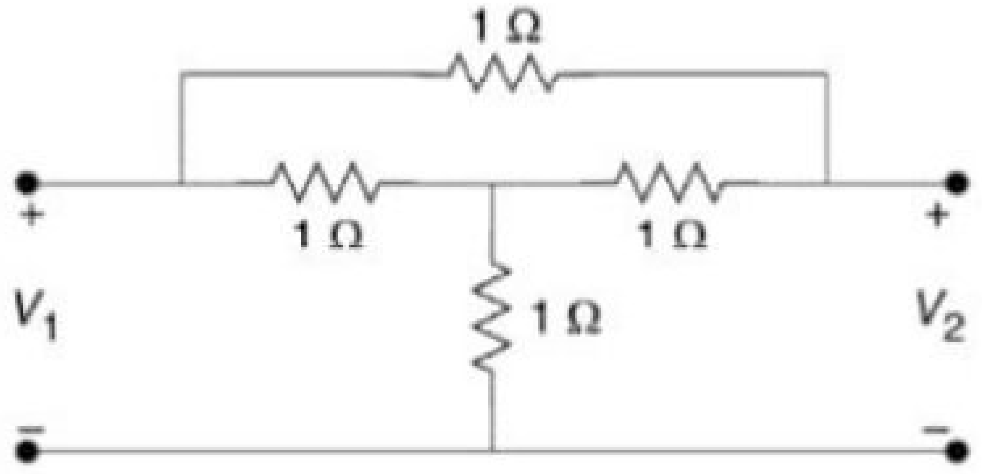
$$\left[\begin{bmatrix} \frac{55}{26} & \frac{50}{13} \\ \frac{7}{20} & 1 \end{bmatrix}\right]$$

7.20 Determine the T-parameters for the network shown in the figure using the concept of interconnection of two two-port networks.



$$\begin{bmatrix} 1+3s^2+s^4 & 3s+4s^3+s^5 \\ 2s+s^3 & 1+3s^2+s^4 \end{bmatrix}$$

7.21 Determine the y parameters of the overall network, considering two networks connected in parallel.



$$\left[y_{11} = y_{22} = \frac{5}{3}\,℧;\ y_{12} = y_{21} = -\frac{4}{3}\,℧\right]$$

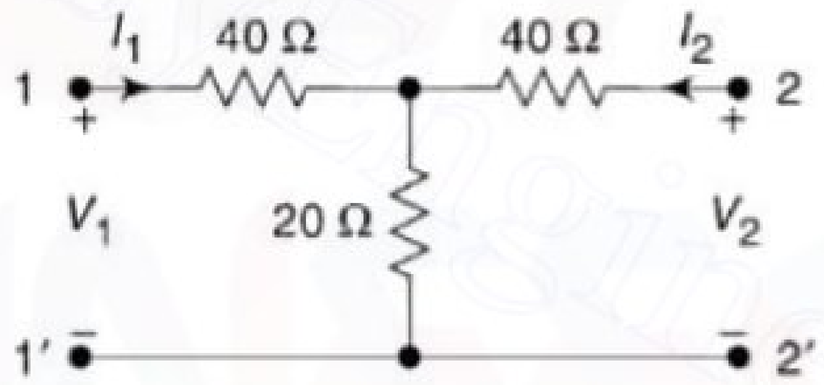
7.22 The z-parameters of a two-port network are

$$z_{11} = 50\ \Omega;\ z_{22} = 30\ \Omega;\ z_{12} = z_{21} = 20\ \Omega$$

Calculate the y-parameters and $ABCD$ parameters of the network.

$$\begin{bmatrix} y_{11} = 0.0273 \text{ mho};\ y_{22} = 0.0454 \text{ mho};\ y_{12} = y_{21} = -0.01818 \text{ mho}; \\ A = 2.5;\ B = 55\Omega;\ C = 0.05 \text{ mho};\ D = 1.5 \end{bmatrix}$$

7.23 For the symmetrical two-port network, calculate the z-parameters and $ABCD$ parameters.



$$[z_{11} = z_{22} = 60\ \Omega;\ z_{12} = z_{21} = 20\ \Omega;\ A = D = 3;\ B = 160\ \Omega;\ C = 0.05 \text{ mho};]$$

SHORT-ANSWER TYPE QUESTIONS

7.1 (a) Consider a linear passive two-port network and explain what are meant by (i) open-circuit impedance parameters and (ii) short-circuit admittance parameters.

(b) What are the open-circuit impedance parameters of a two-port network? How can the transmission parameters be obtained from open-circuit impedance parameters?

(c) Establish, for two-port networks, the relationship between the transmission parameters and the open-circuit parameters.

(d) Define z- and y-parameters of a typical four terminal network. Determine the relationship between the z and y parameters.

(e) Express h-parameters in terms of z-parameters for a two-port network.

(f) Derive expressions for the y-parameters in terms of $ABCD$ parameters of a two-port network.

7.2 (a) What do you understand by a reciprocal network? What is a symmetrical network?

(b) Write technical note on derivation of short-circuit admittance parameter y_{12} of a symmetrical and reciprocal two-port lattice network.

(c) How will you find the π-equivalent of a given network when its y-parameters are known?

7.3 (a) Explain what are meant by the transmission ($ABCD$) parameters of a two-port network. Derive the conditions necessary to be satisfied for the network to be (i) reciprocal and (ii) symmetrical.

Or,

Prove that for a reciprocal two-port network,

$$\Delta T = (AD - BC) = 1$$

(b) Prove that for a symmetrical two-port network,

$$\Delta h = (h_{11}h_{22} - h_{12}h_{21}) = 1$$

7.4 (a) Two two-port networks are connected in parallel. Prove that the overall y-parameters are the sum of corresponding individual y-parameters.

(b) Two two-port networks are connected in cascade. Prove that the overall transmission parameter matrix is the product of individual transmission parameter matrices.

(c) Two two-port networks are connected in series. Prove that the overall z-parameters are the sum of corresponding individual z-parameters.

7.5 What are transmission parameters? Where are they most effectively used? Establish, for two-port networks, the relationship between the transmission parameters and the open circuit impedance parameters.

ANSWERS TO MULTIPLE-CHOICE QUESTIONS

7.1 (d)	7.2 (c)	7.3 (a)	7.4 (a)	7.5 (d)	7.6 (c)	7.7 (b)
7.8 (d)	7.9 (a)	7.10 (b)	7.11 (d)	7.12 (d)	7.13 (c)	7.14 (c)
7.15 (c)	7.16 (b)	7.17 (b)	7.18 (a)	7.19 (d)	7.20 (d)	7.21 (b)
7.22 (a)	7.23 (b)	7.24 (d)	7.25 (a)	7.26 (b)	7.27 (d)	7.28 (d)
7.29 (a)	7.30 (c)	7.31 (a)	7.32 (c)	7.33 (d)	7.34 (d)	7.35 (a)
7.36 (b)	7.37 (a)	7.38 (d)	7.39 (d)	7.40 (d)	7.41 (a)	7.42 (c)
7.43 (a)	7.44 (d)	7.45 (b)	7.46 (d)	7.47 (b)	7.48 (d)	